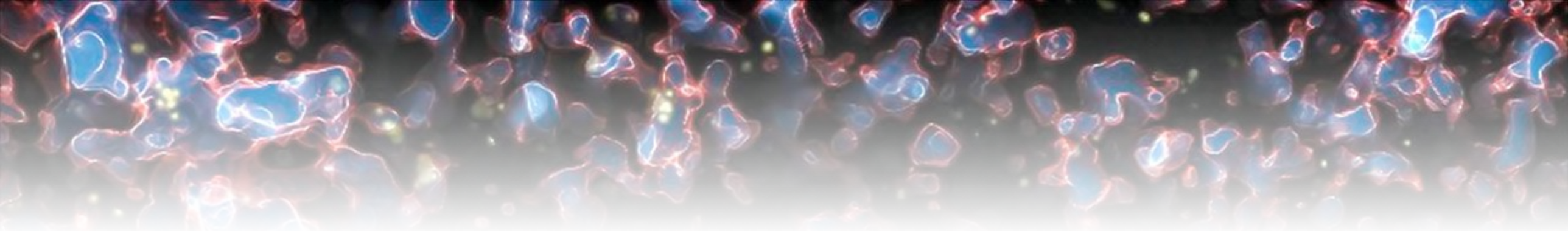

Pushing Inference to the Sky: A Differentiable Forward Model for the 21 cm Global Signal (work in progress..)

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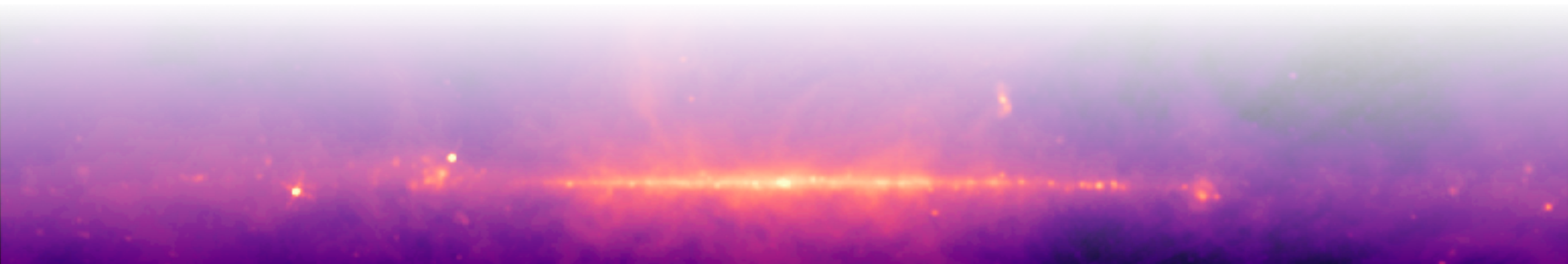
Berkeley Global Signal Conference

MIT KAVLI INSTITUTE



1. How does performing inference on the sky help?

- More faithful accounting of beam, FGs, and their degeneracies

- Fully exploit correlations in the data between LSTs

2. Is detection theoretically possible?

- Even in ideal circumstances, 21 cm global signal extraction is **highly degenerate** with FG spectral modes (c.f. Liu et al. 2012)

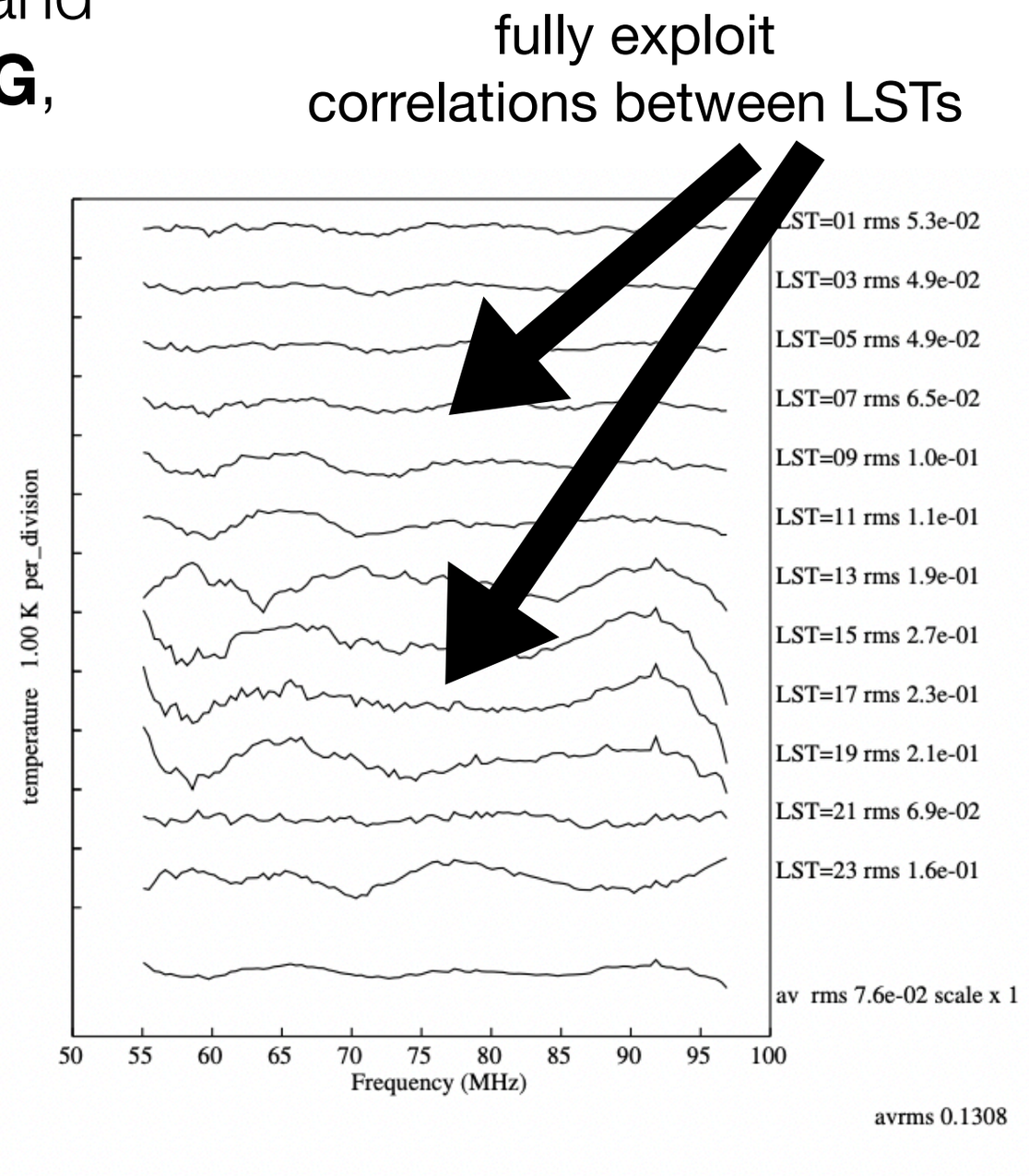
- Forward modeling of **flexible** but **regularized** sky + instrument models is key to a robust 21 cm GS constraint

Why an end-to-end forward model?

• Robust signal extraction must understand the joint posterior between the **beam**, **FG**, and **21 cm** signal

• This requires a data model that **starts on the sky**, as opposed to a spectrum-based data model

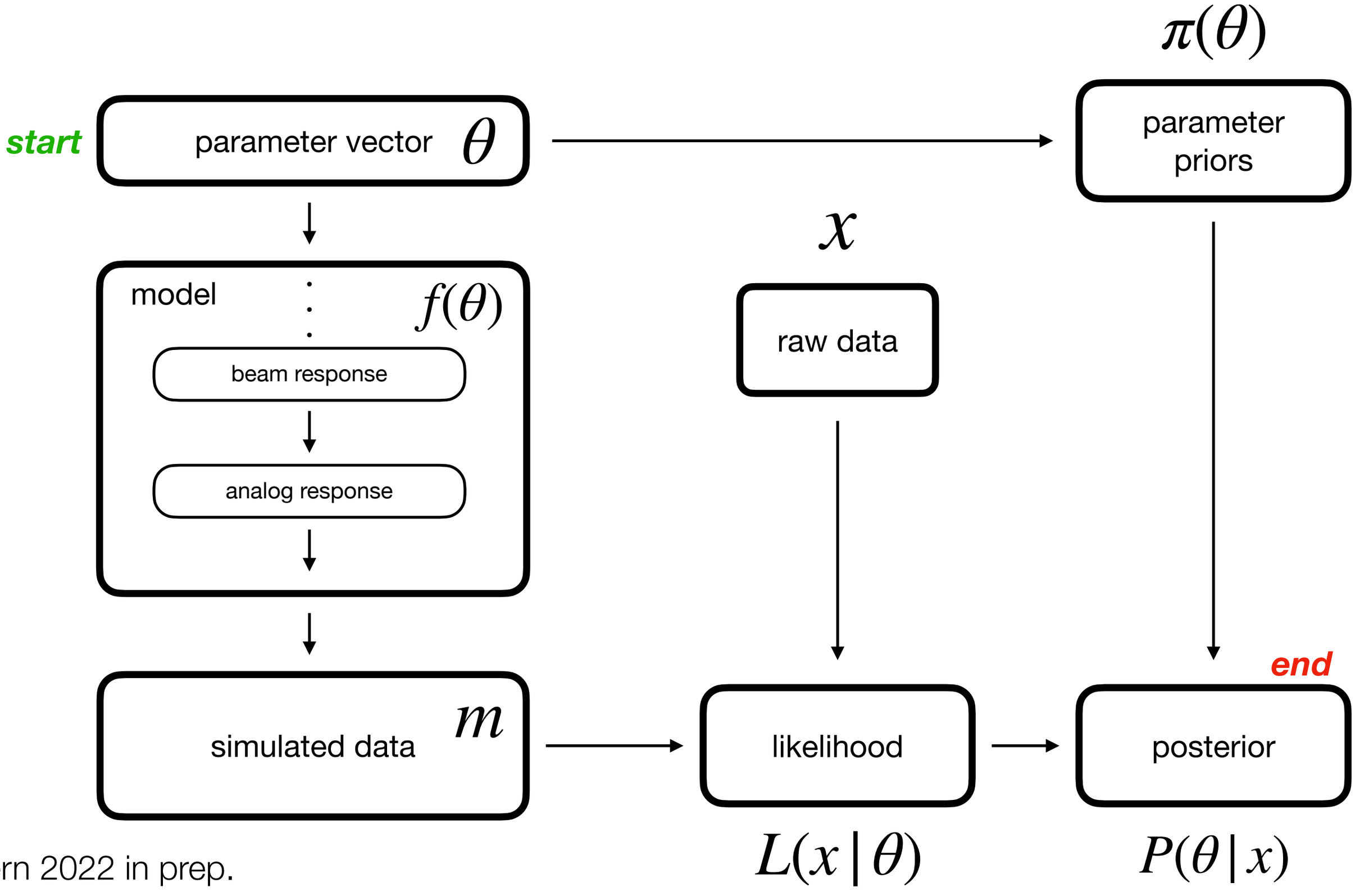
• Including the visibility sky integral in our data model allows us to fully (and properly) exploit correlations between LSTs



See also Tauscher+, Nhan+, Rapetti+, Anstey+

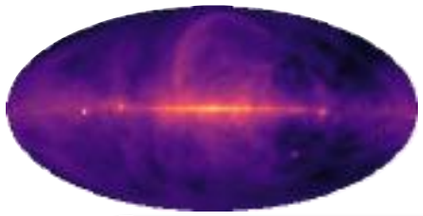
A. Rodgers Memo #374 2021

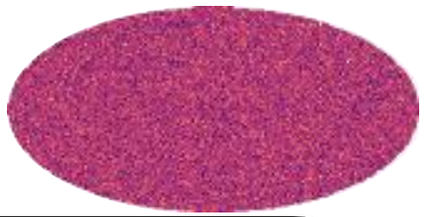
What is a (Bayesian) forward model?

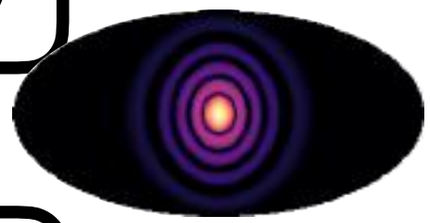


Kern 2022 in prep.

What is a (Bayesian) forward model?







start

parameter vector θ

$\pi(\theta)$

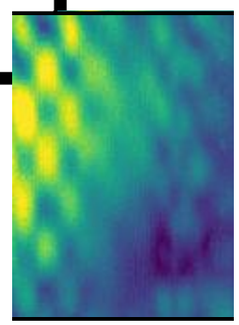
parameter priors

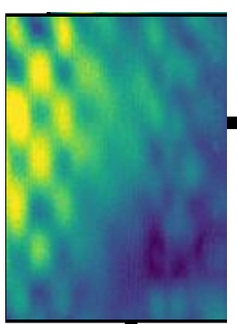
x

model $f(\theta)$

- beam response
- analog response

raw data





simulated data m

likelihood

$L(x \mid \theta)$

end

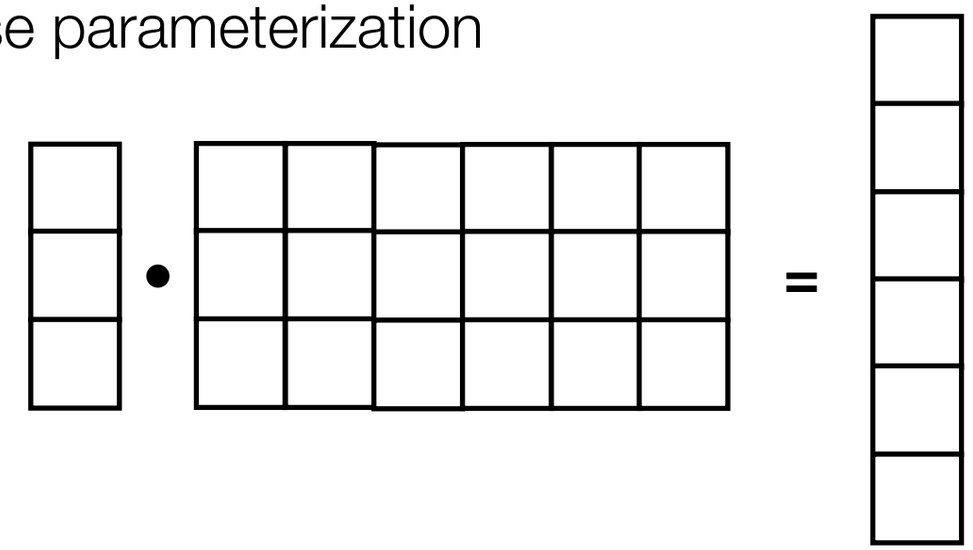
posterior

$P(\theta \mid x)$

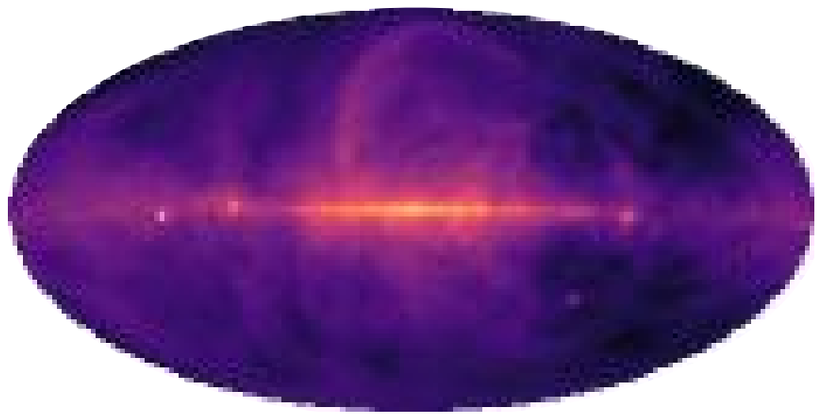
Kern 2022 in prep.

What is a (Bayesian) forward model?

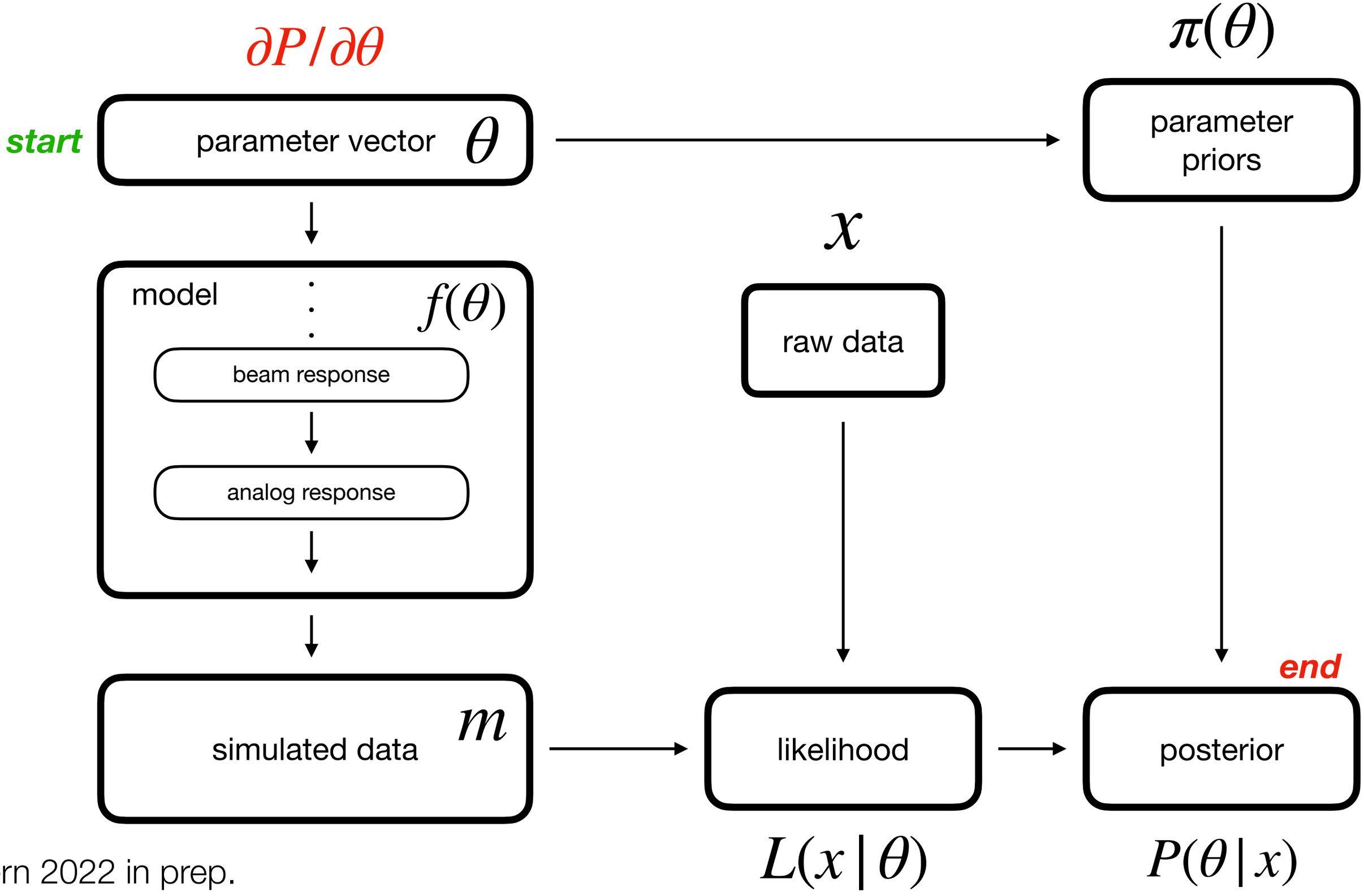
sparse parameterization



$$a_{lm} \cdot Y_{lm} =$$

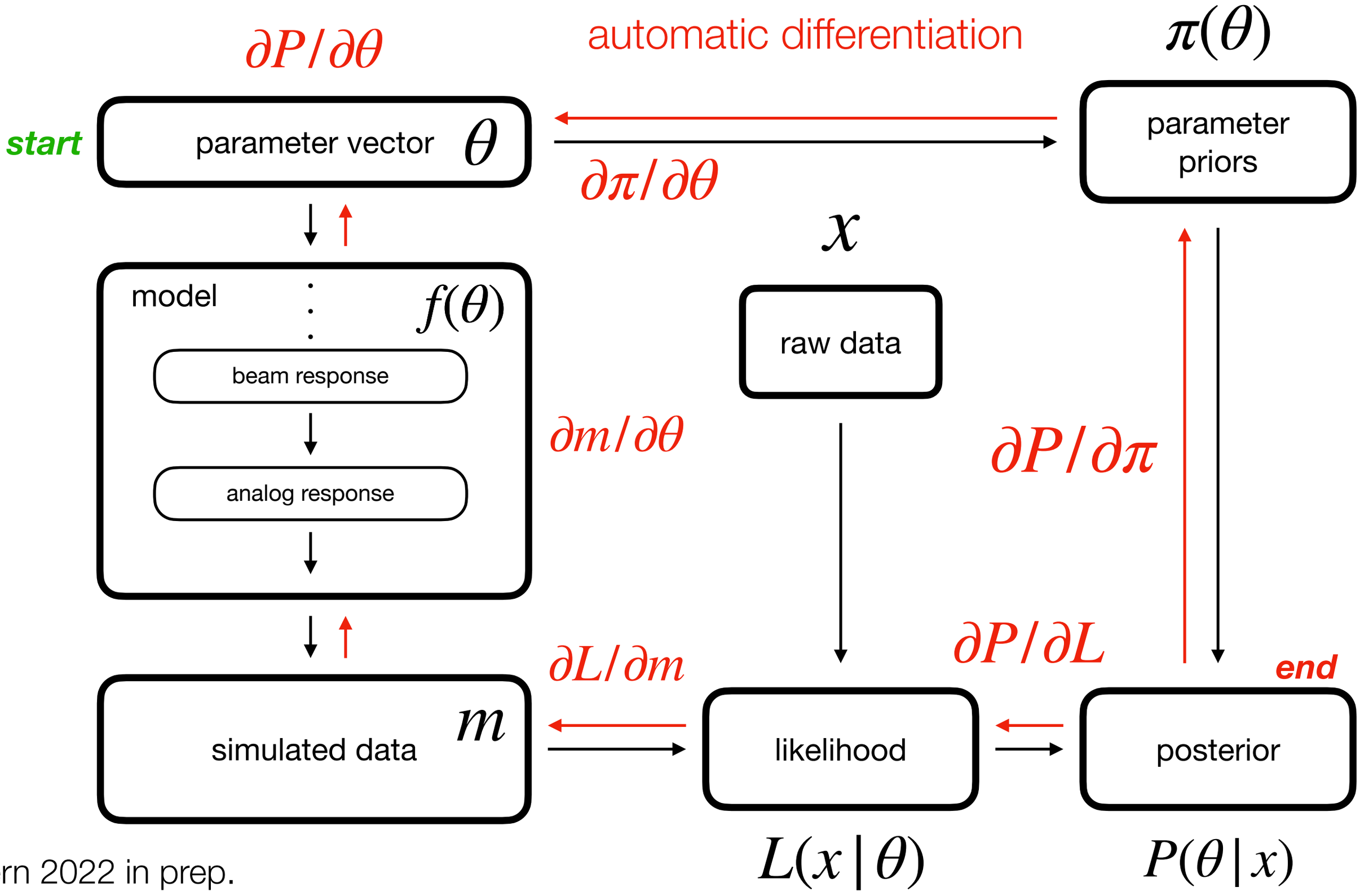


How to make it differentiable?



Kern 2022 in prep.

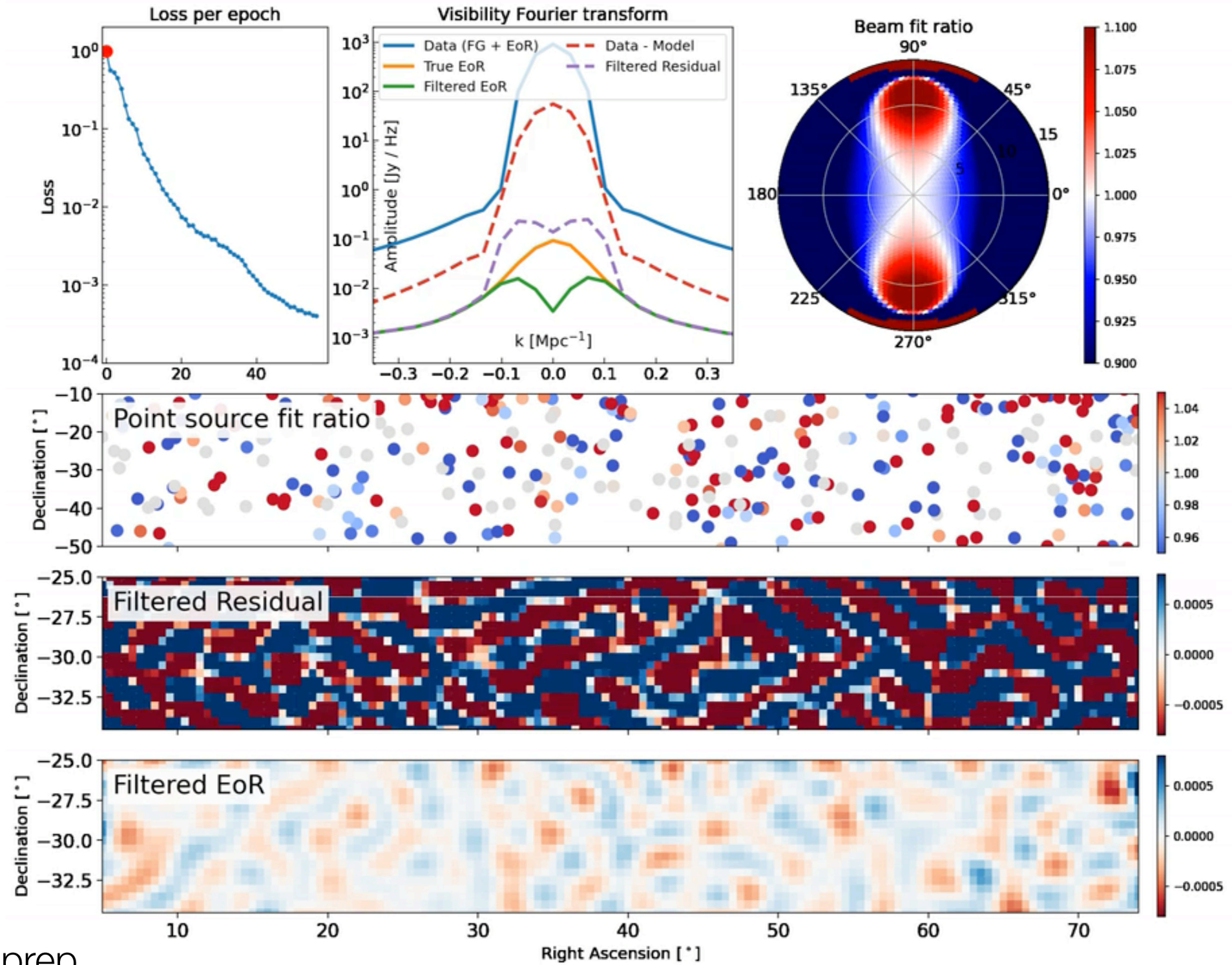
How to make it differentiable?



Kern 2022 in prep.

Example with an interferometer

point source + beam optimization



Kern 2022 in prep.

For now: testing degeneracies

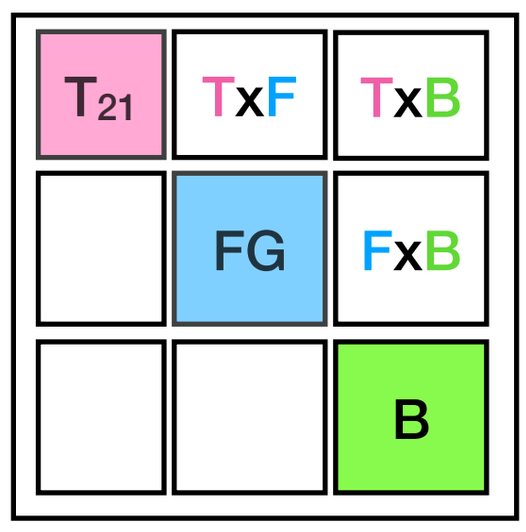
For now: testing degeneracies

Posterior expansion via the Fisher matrix (aka the -Hessian)

$$F_{ij} = -\frac{\partial^2 \ln P}{\partial\theta_i \partial\theta_j}$$

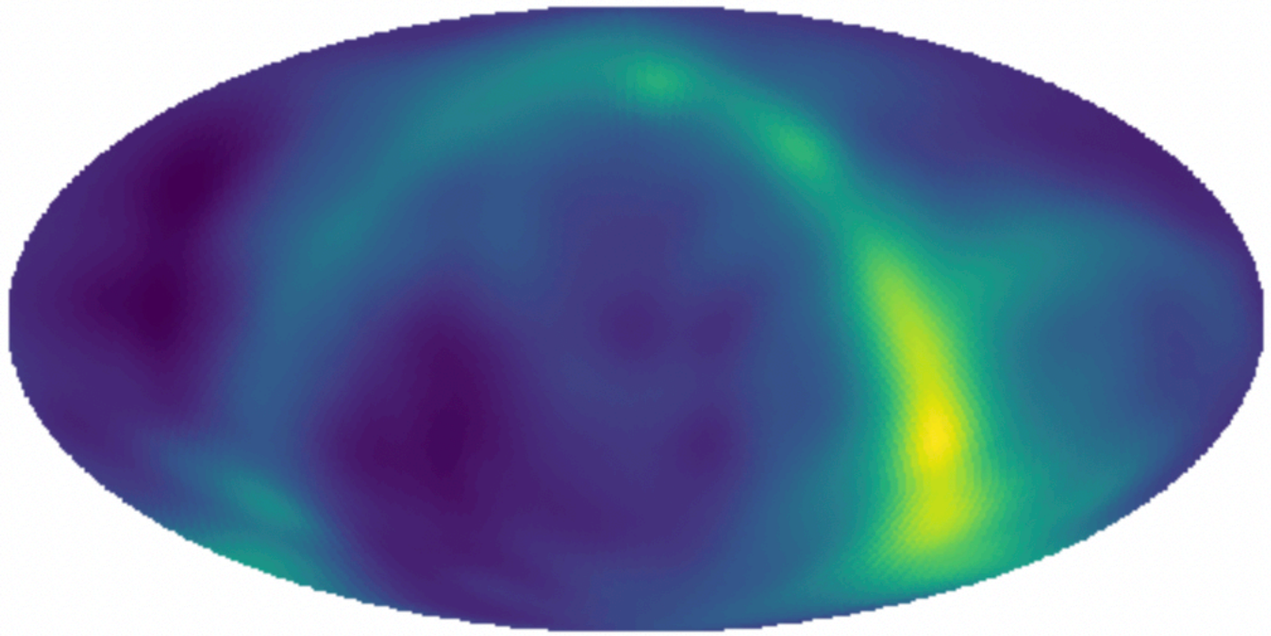
Can compute the Hessian exactly via automatic diff.

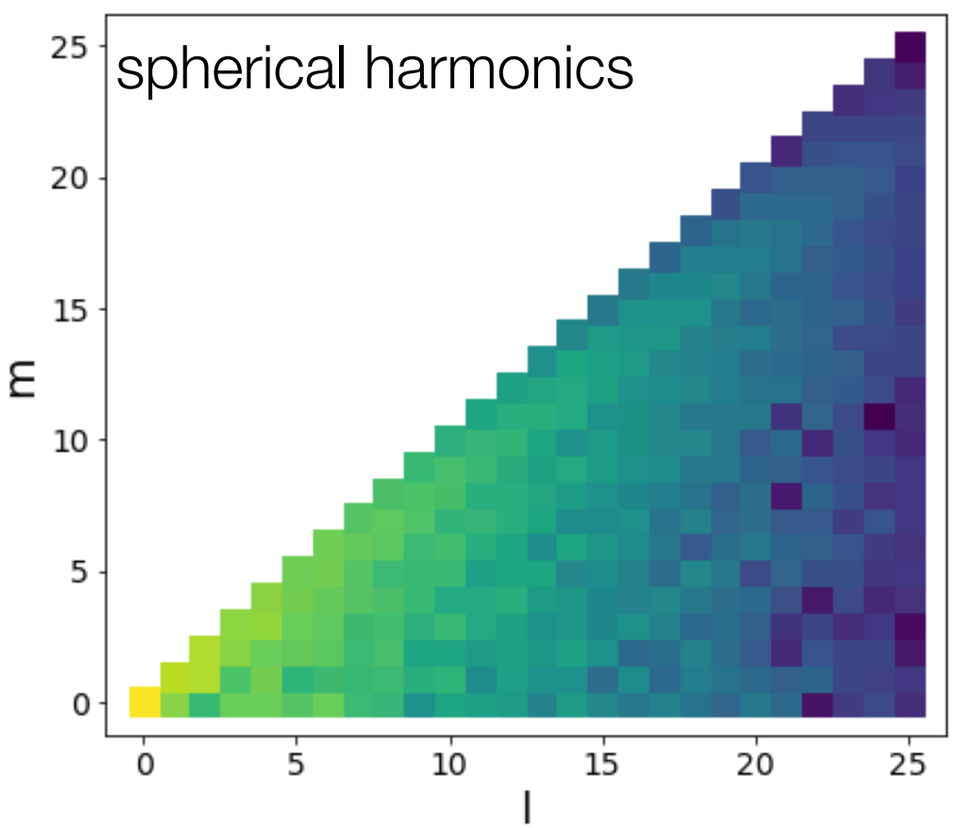
$$C_{ij} = [\boldsymbol{F}^{-1}]_{ij}$$

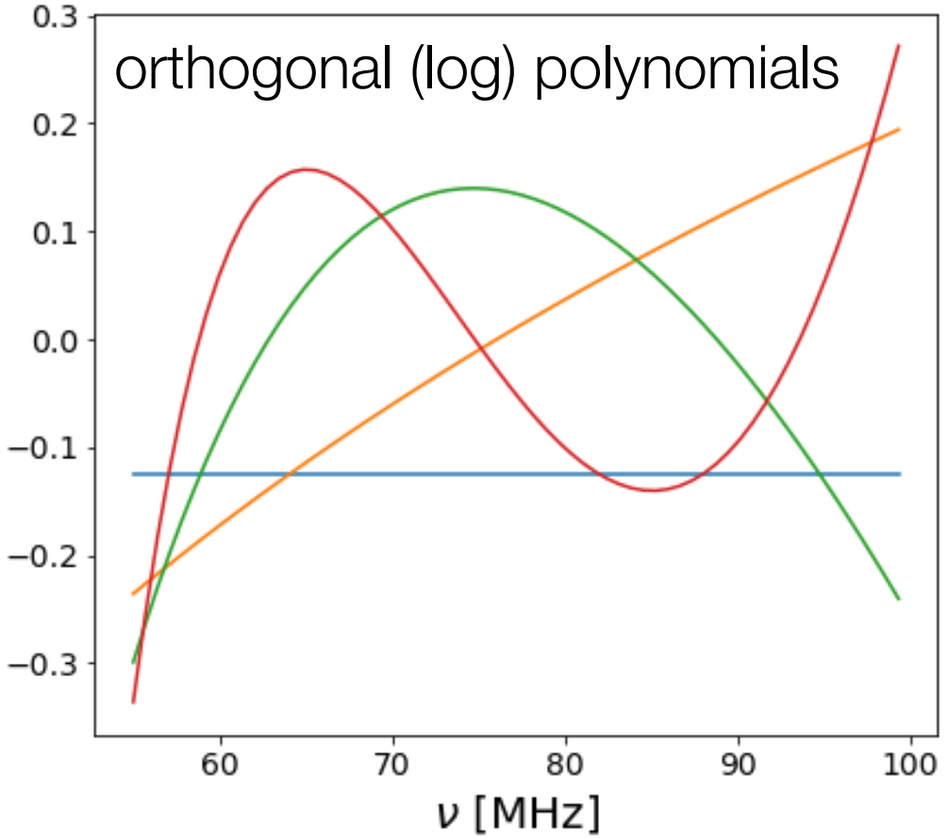


The full F inverse accounts for degeneracies between T21, FG, and beam.
No noise and no front-end calibration in these tests.

Foreground parameterization

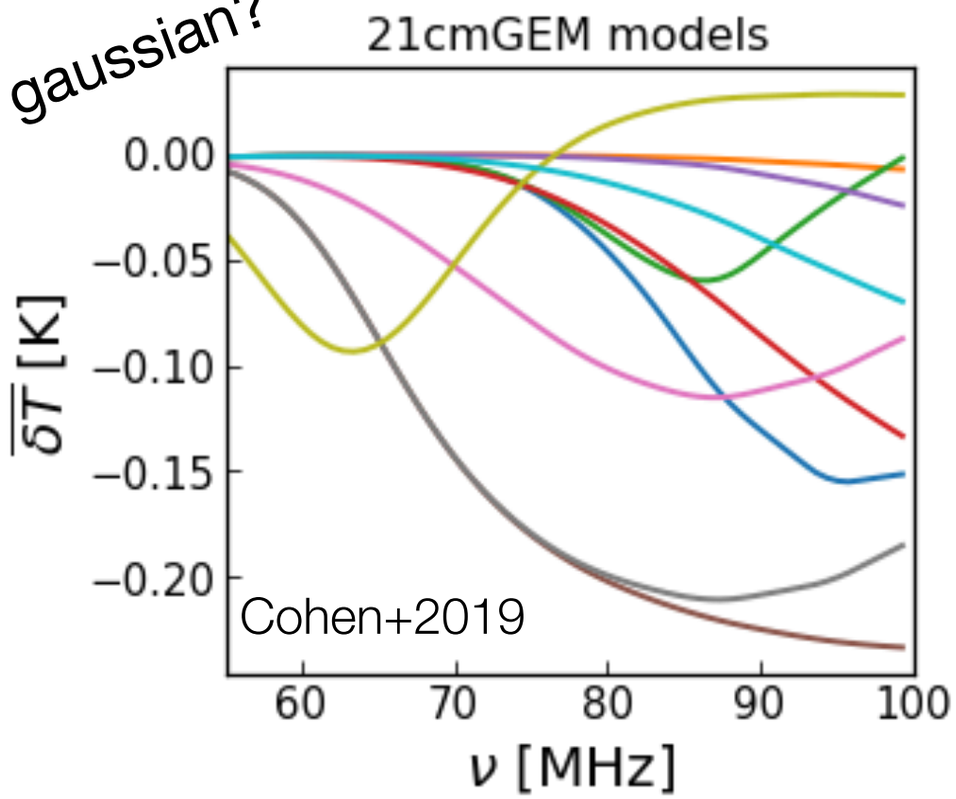


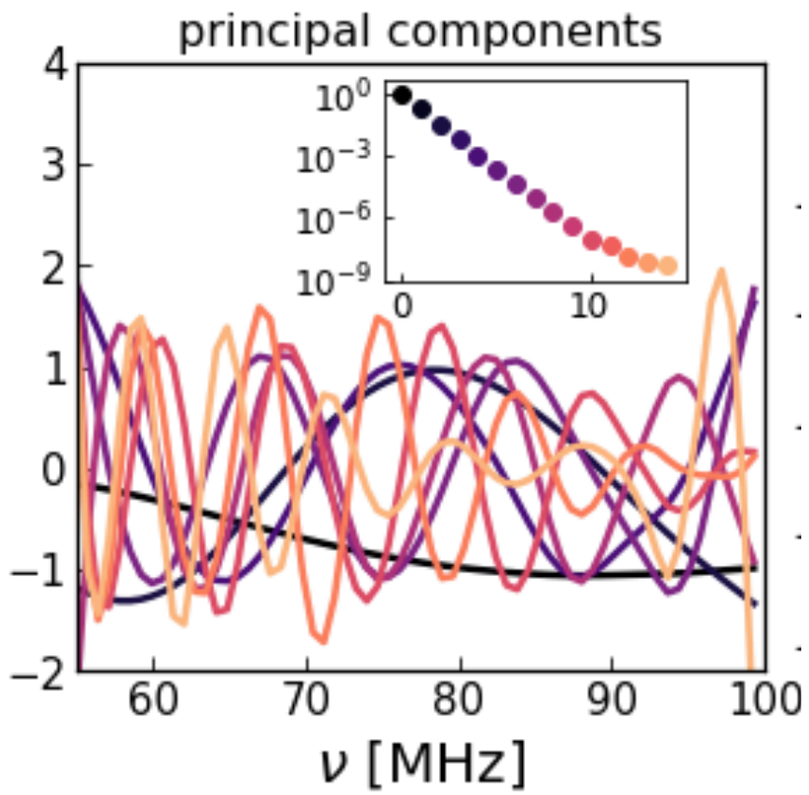


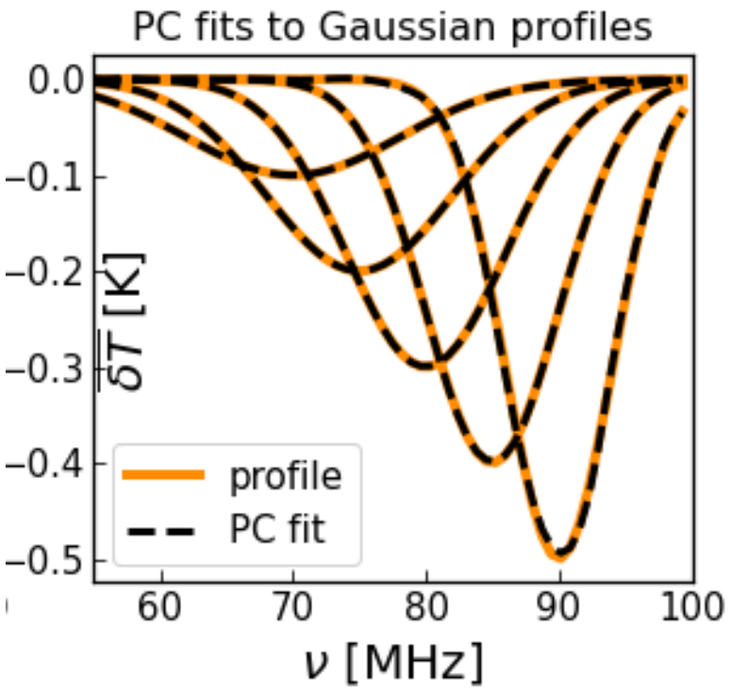


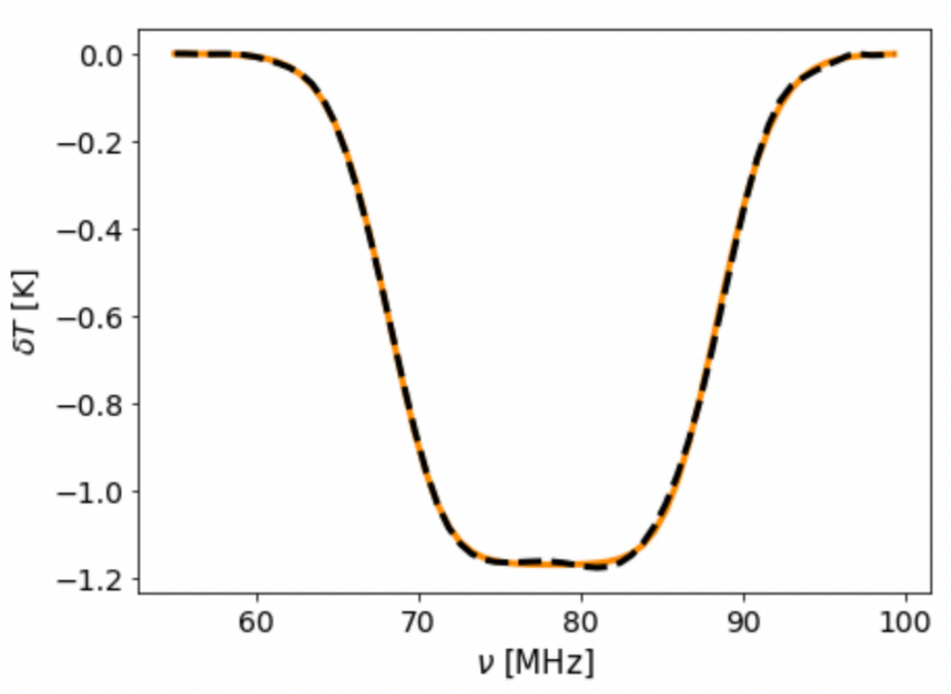
21 cm parameterization

flattened gaussian?

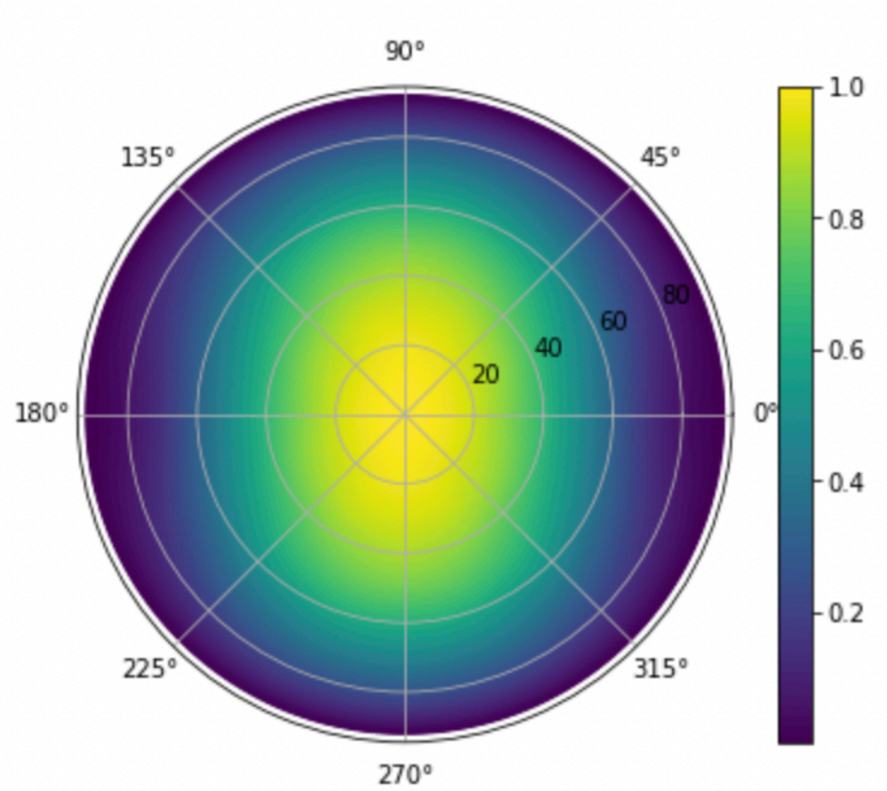


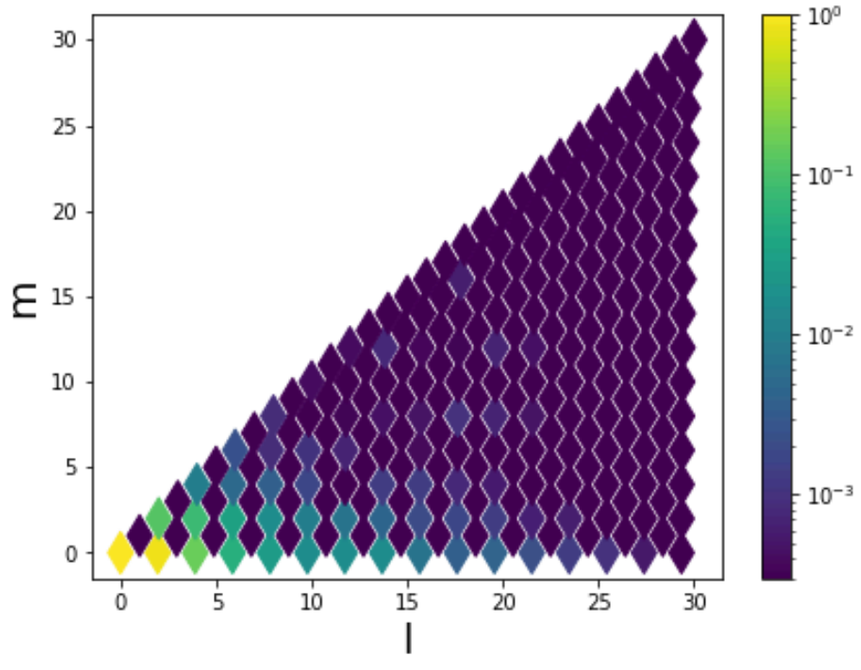


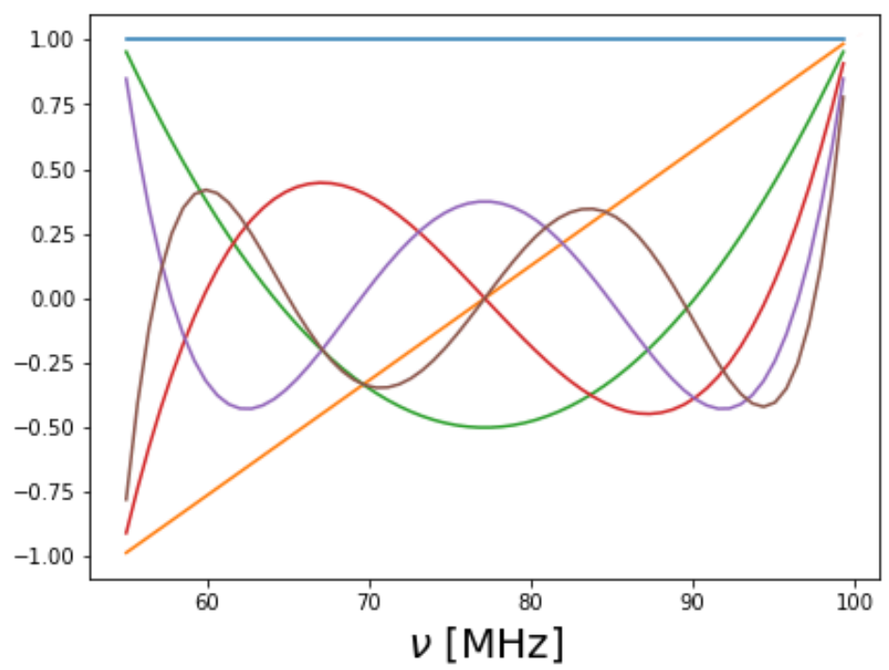




Instrument parameterization: EDGES beam

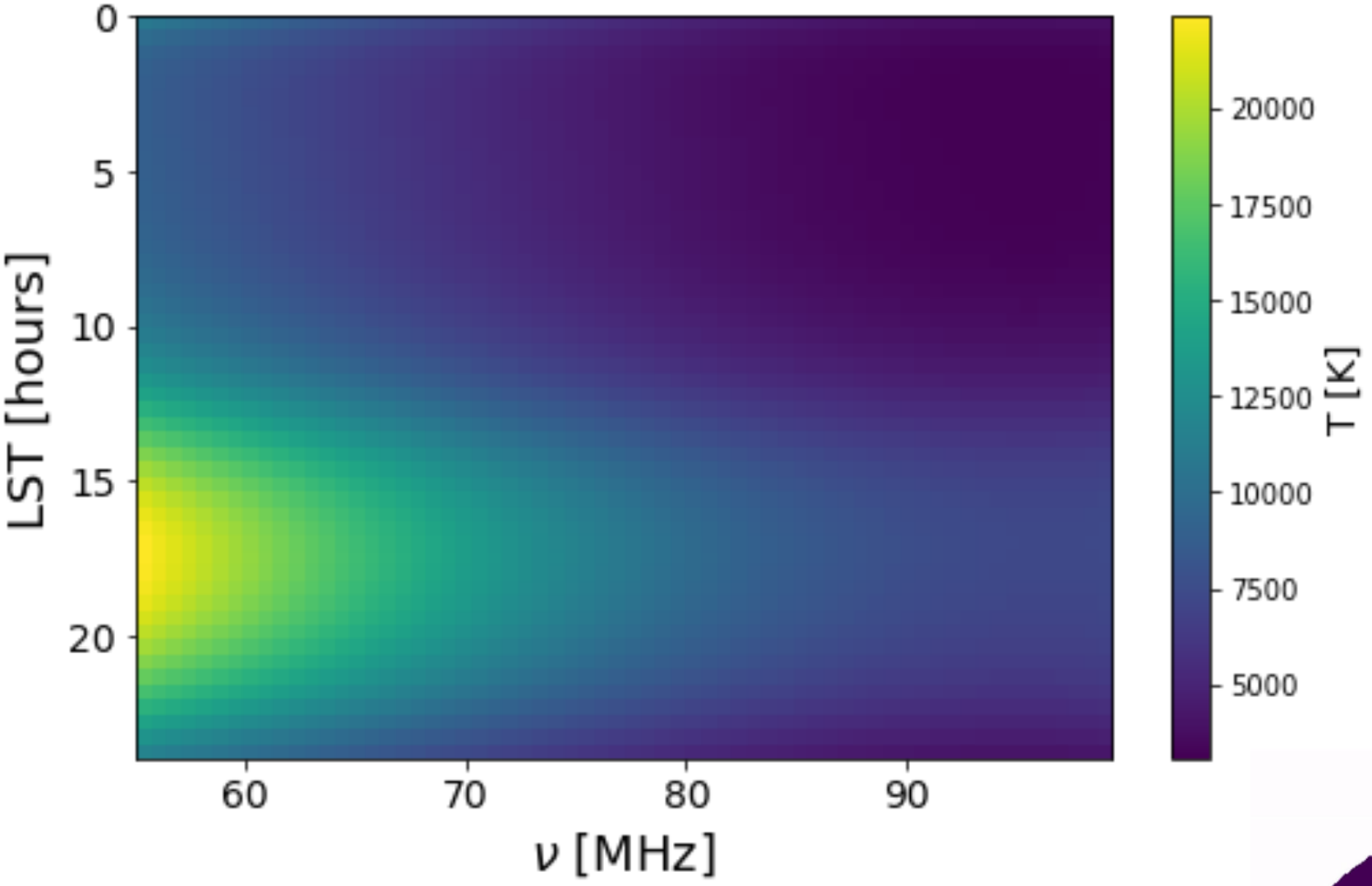




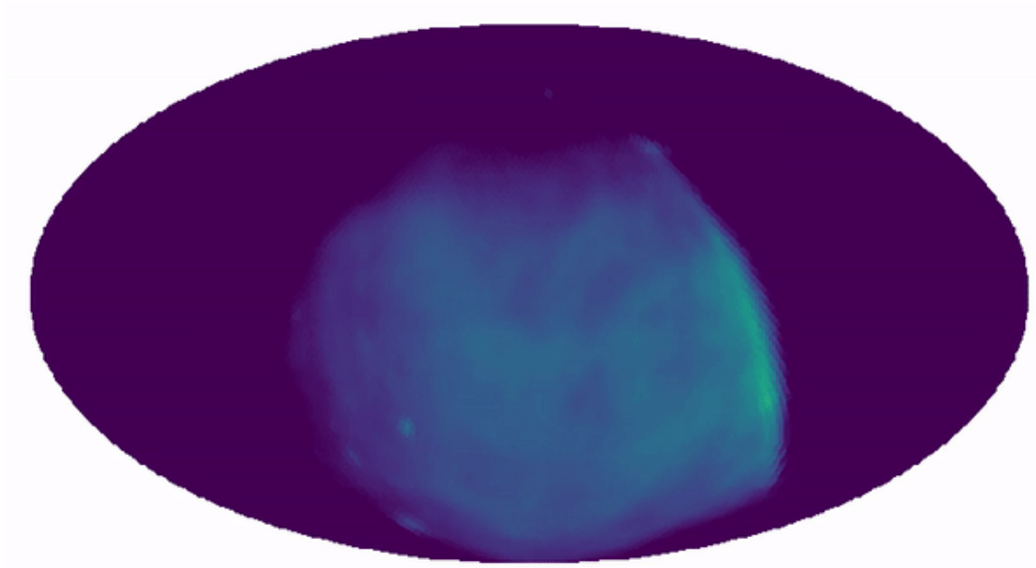


Courtesy A. Rodgers & Mahesh+2021

Mock observation setup

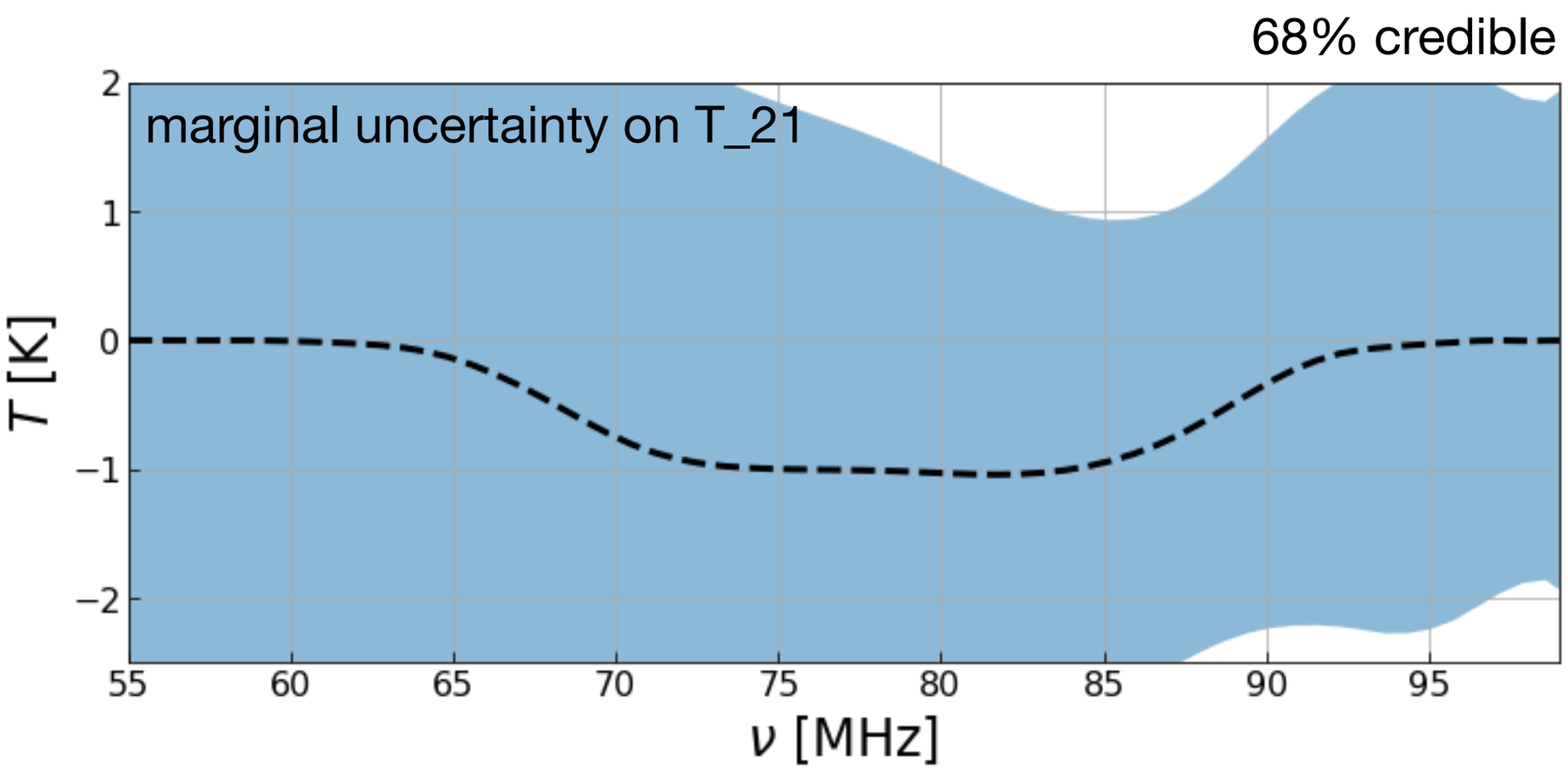


Nfrequency = 64
Ntimes = 50
Npixel = 12288 (NSIDE=32)
no noise, no calibration



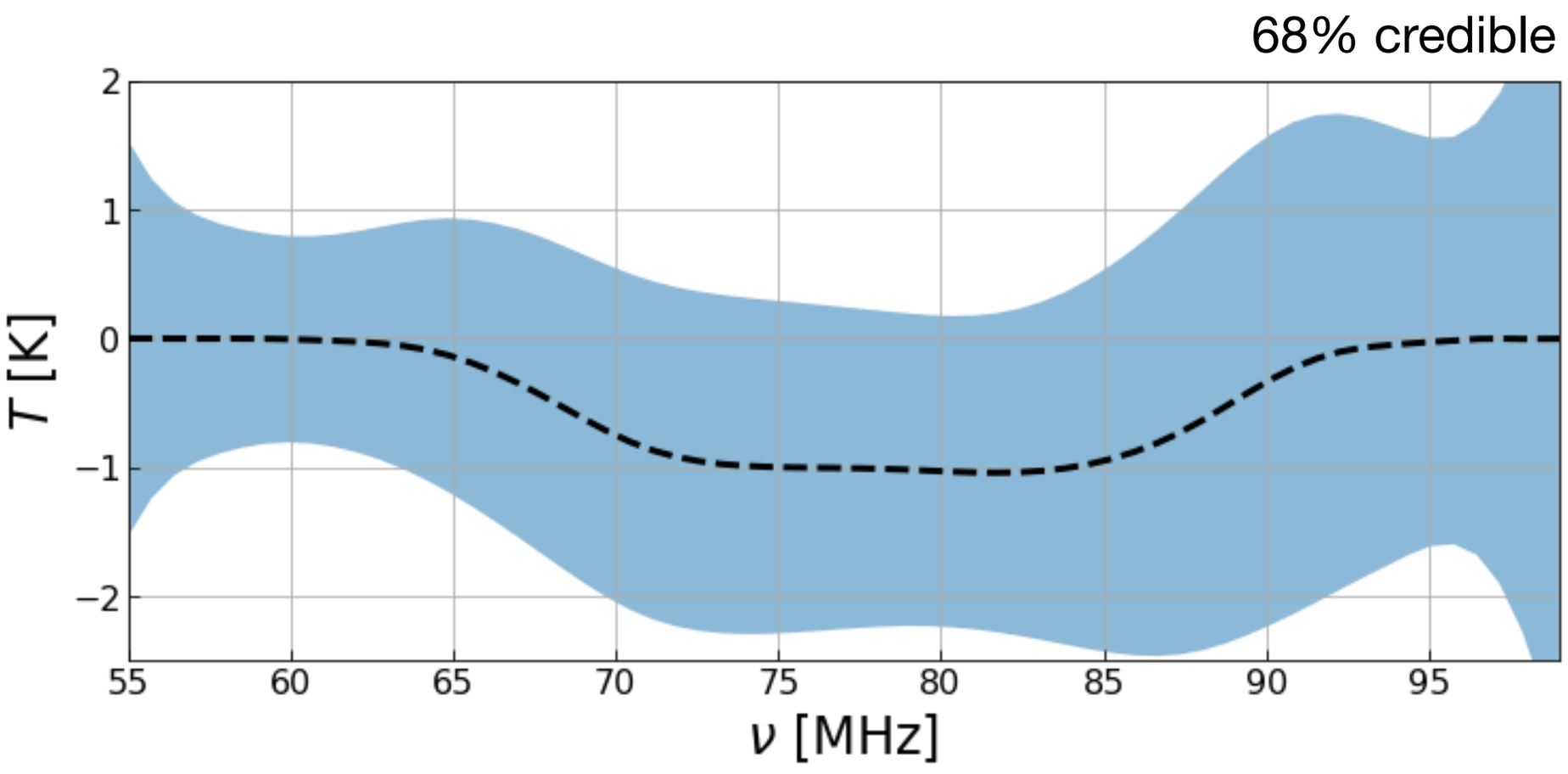
Testing for degeneracies

- **T_21** (no prior), **Beam** (no prior), FG (known)
- 1 LST



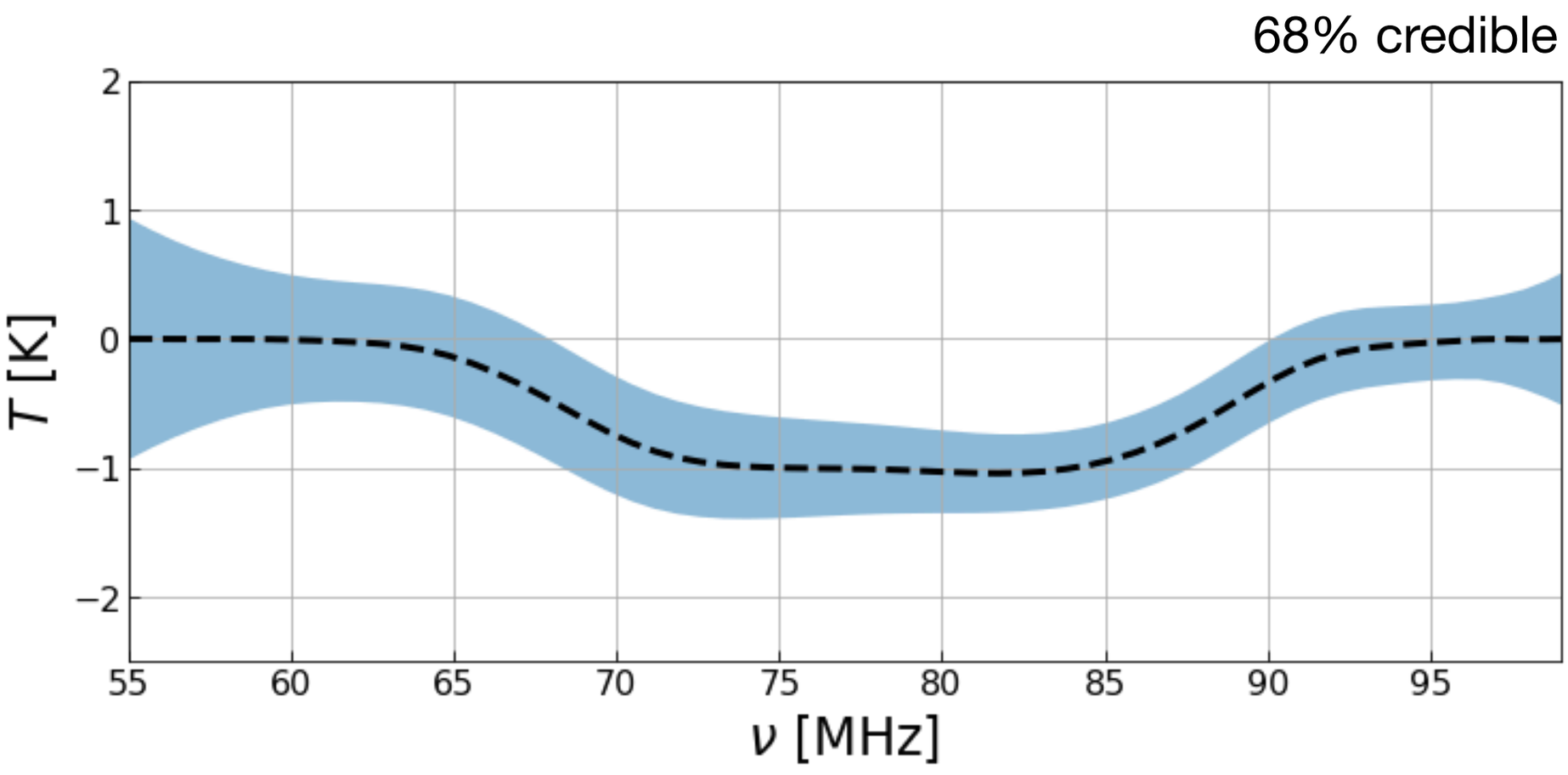
Testing for degeneracies

- **T_21** (2 K prior), **Beam** (1% prior), FG (known)
- 1 LST



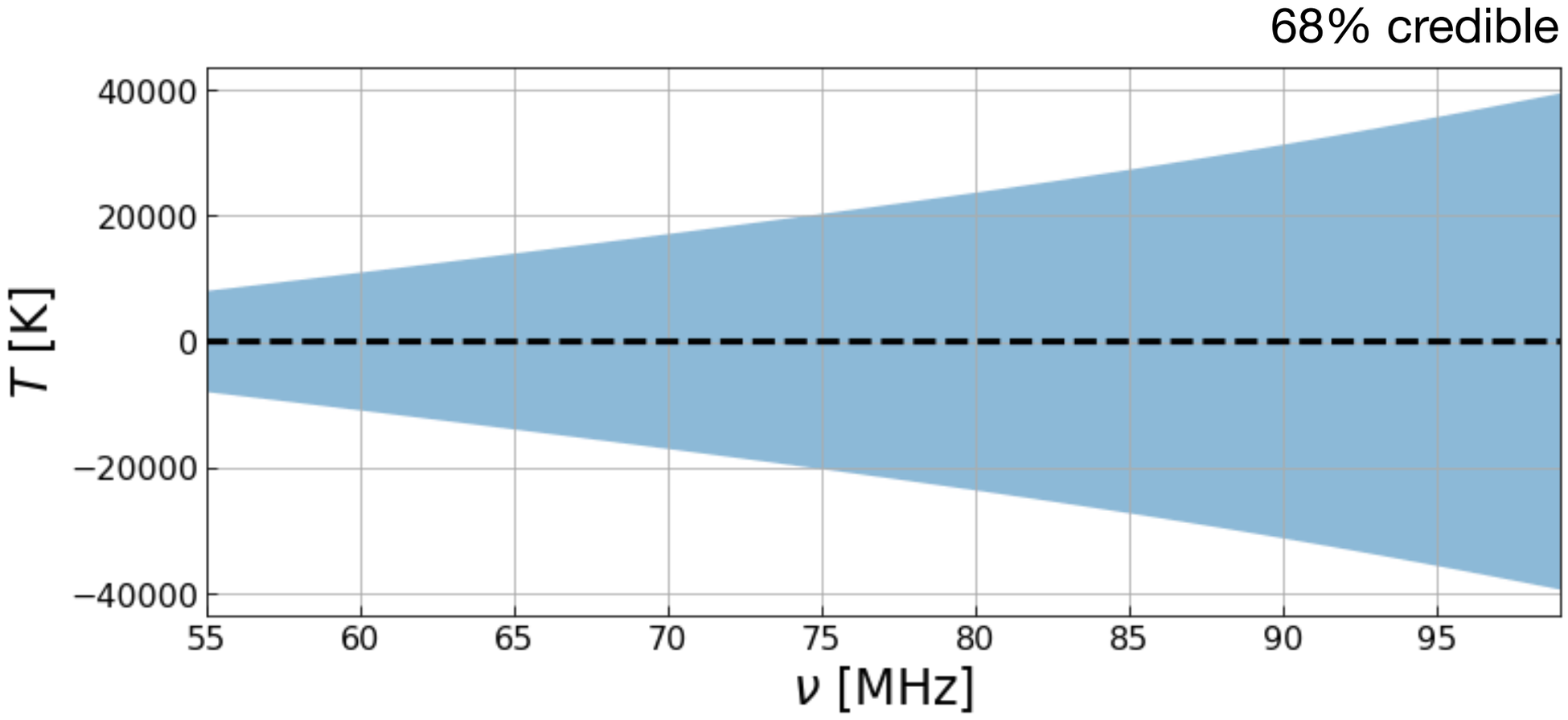
Testing for degeneracies

- **T_21** (2 K prior), **Beam** (1% prior), FG (known)
- 24 hours LST



Testing for degeneracies

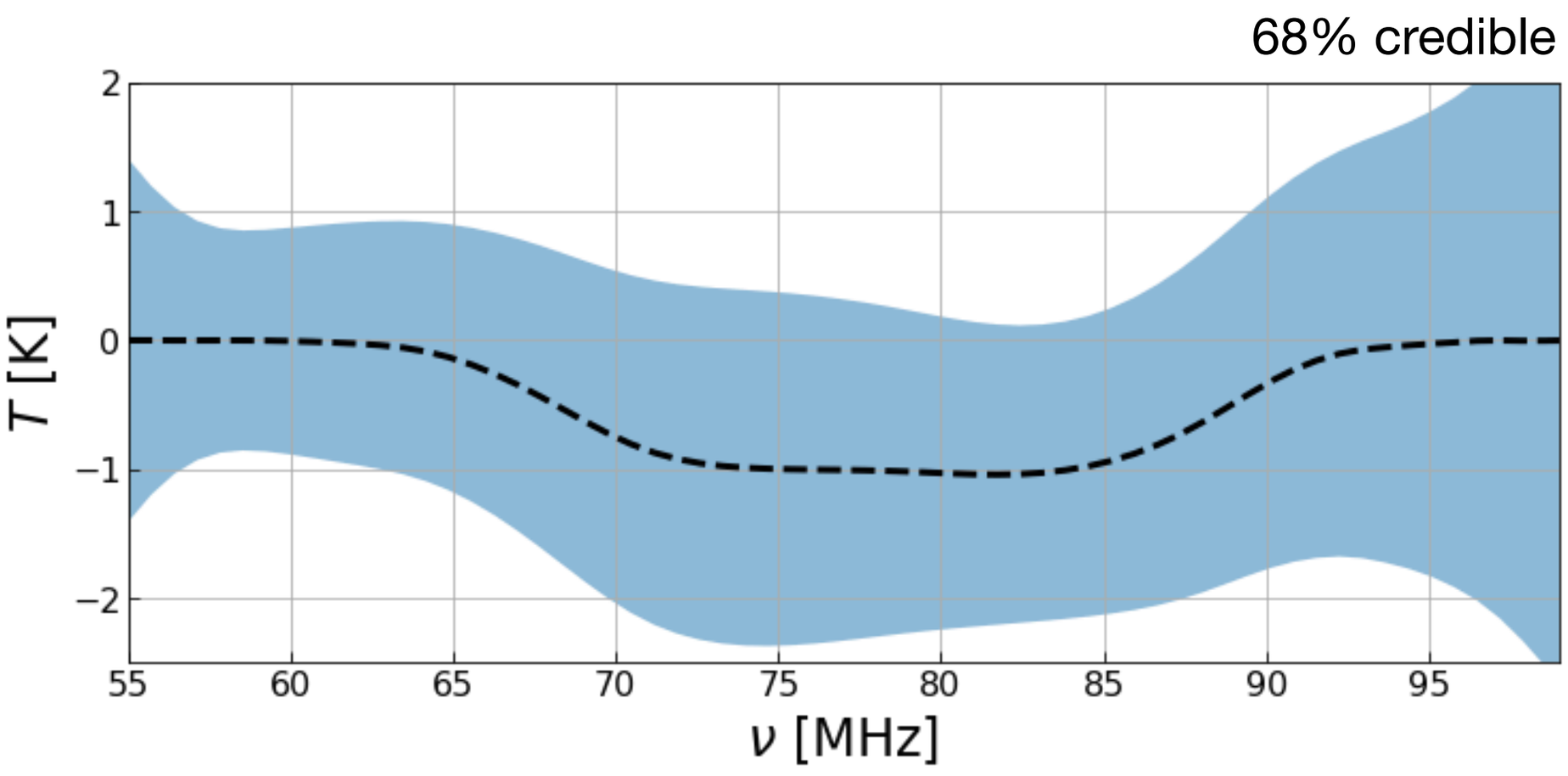
- **FG** (no prior), **T_21** (no prior), Beam (known)
- 24 hours LST



See also Liu+2012

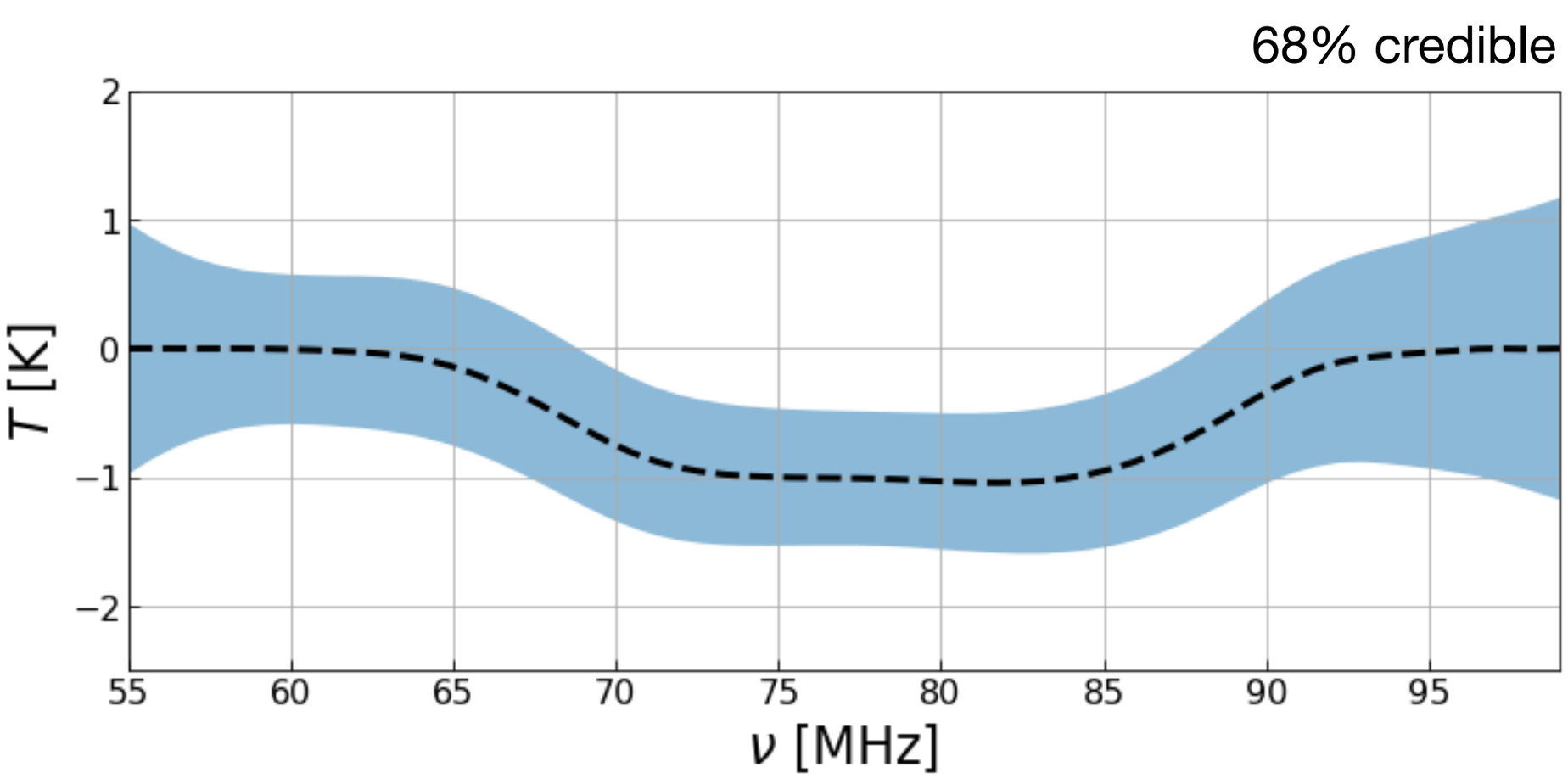
Testing for degeneracies

- **FG** (10% prior), **T_21** (2 K prior), Beam (known)
- 24 hours LST



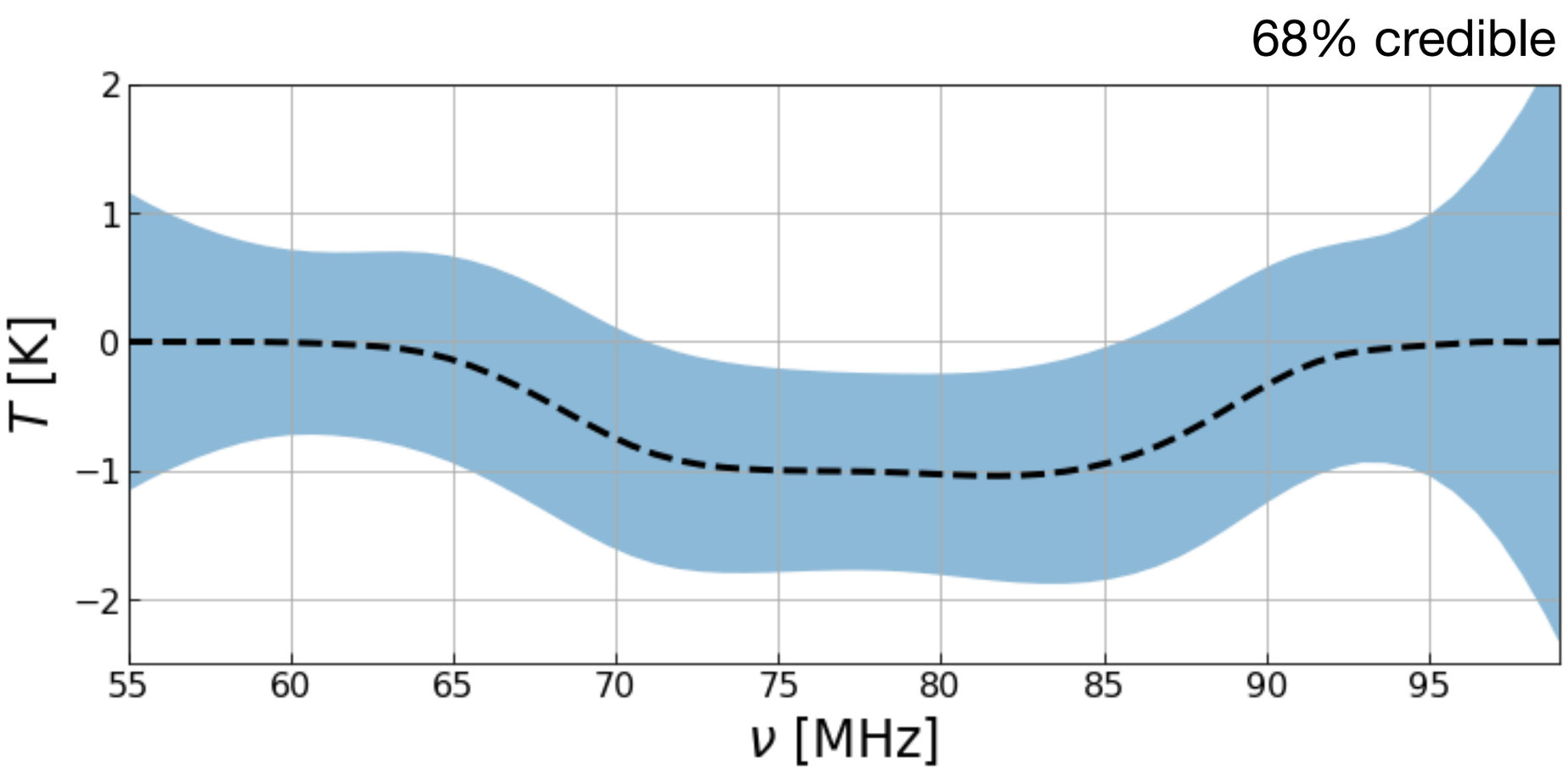
Testing for degeneracies

- **FG** (10% prior, m>0), **T_21** (2 K prior), Beam (known)
- 24 hours LST



Testing for degeneracies

- **FG** (10% prior, m>0), **T_21** (2 K prior), **Beam** (1% prior)
- 24 hours LST



Testing for degeneracies

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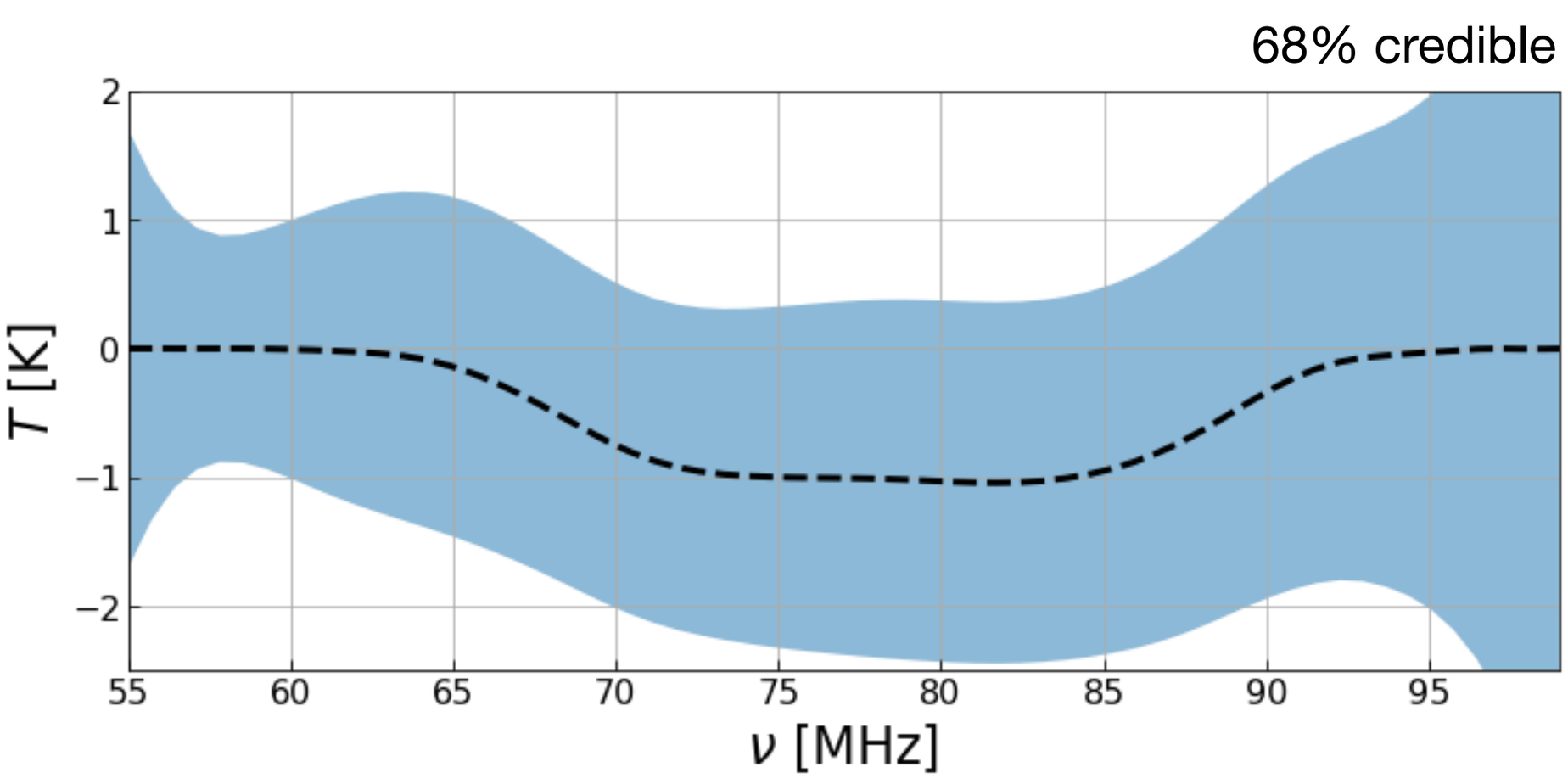
- 1 LST



Some avenues for progress?

- Down-weight m=0 angular modes with a FG spatial covariance (e.g. Liu et al. 2012)

  Do we even know this covariance accurately?

  Requires more complex observations (smaller beams, steerable pointing)

- Set a prior on the global signal with a P(k) detection

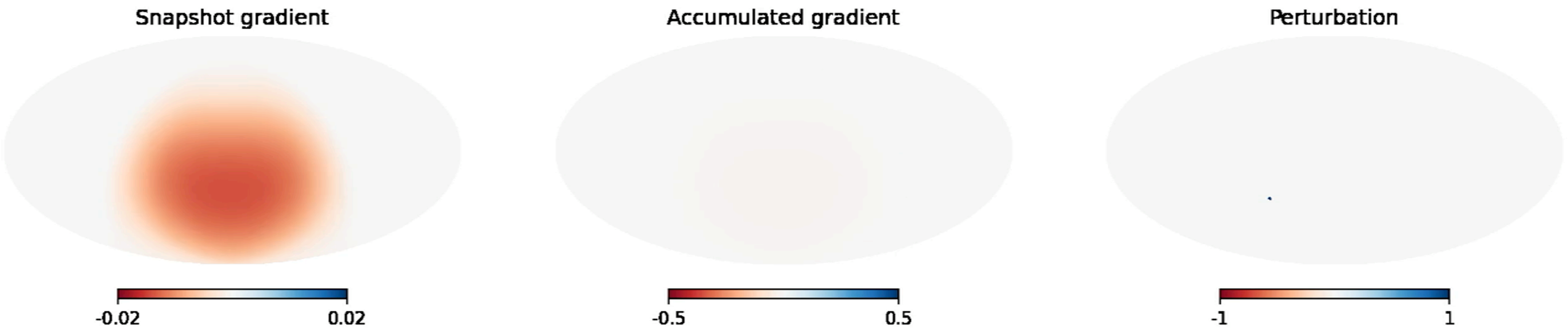
  This is model dependent

Summary

- Forward modeling is key to a deeper understanding of **degeneracies** between the signal and systematics, and for fully leveraging the **statistical power** in the data (e.g. multi-LSTs). More results on optimization and sampling to come soon…

- Even in ideal circumstances (i.e. perfect front-end calibration, perfect beam knowledge, and multi-LSTs), “detection” of a 21 cm global signal is complicated by degenerate FG modes

Cool features: gradient maps



Performance