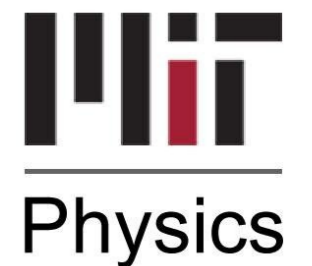


Revisiting Gaussian Process Foreground Subtraction for 21 cm Cosmology

Nick Kern

Pappalardo Postdoctoral Fellow
MIT Kavli Institute

Science at Low Frequencies VII
December 2, 2020



Revisiting the GPR technique

arXiv.org > astro-ph > arXiv:2010.15892

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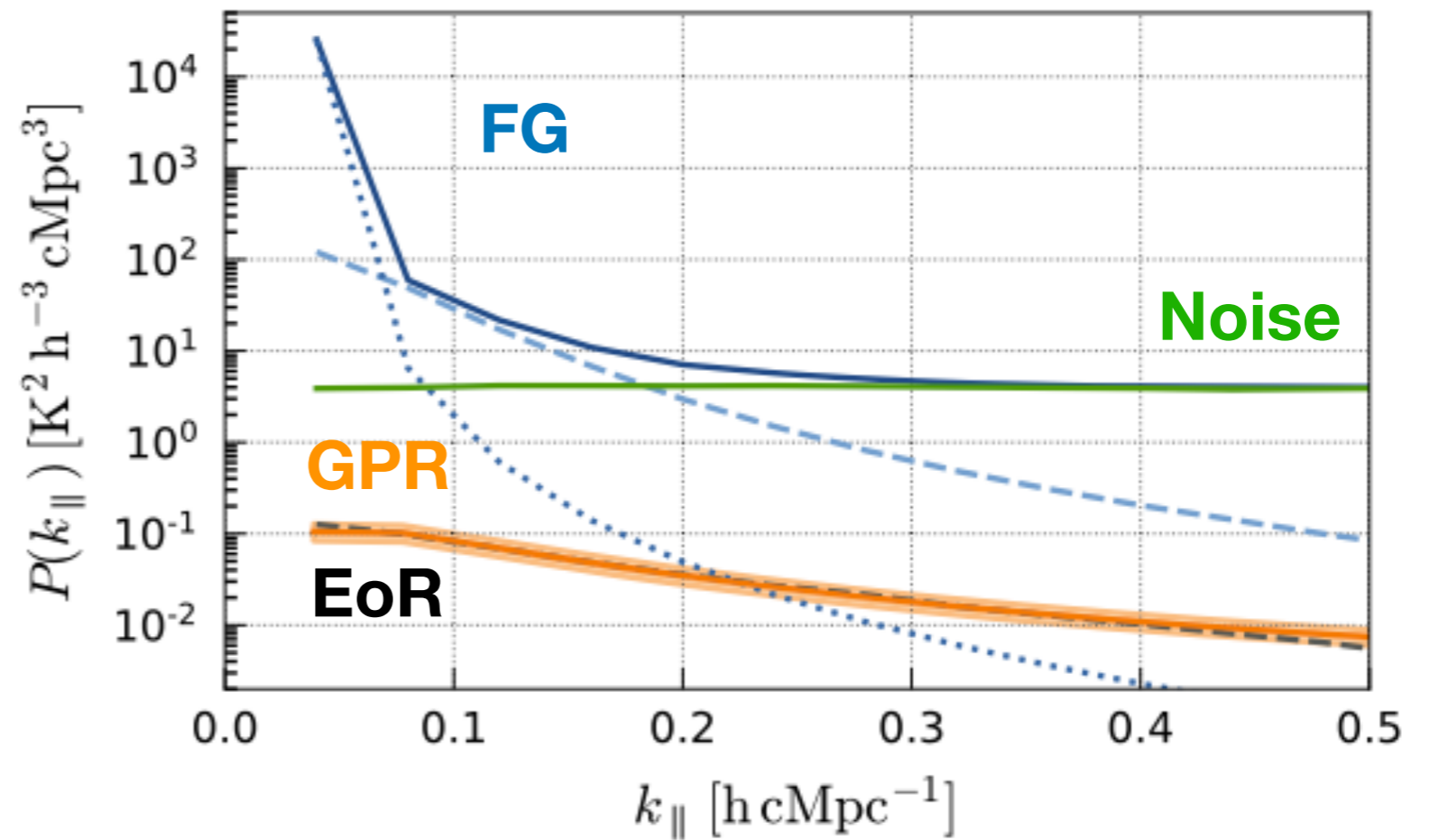
[Submitted on 29 Oct 2020]

Gaussian Process Foreground Subtraction and Power Spectrum Estimation for 21 cm Cosmology (GPR-FS)

Nicholas Kern, Adrian Liu

Gaussian process regression for FG subtraction (GPR-FS)

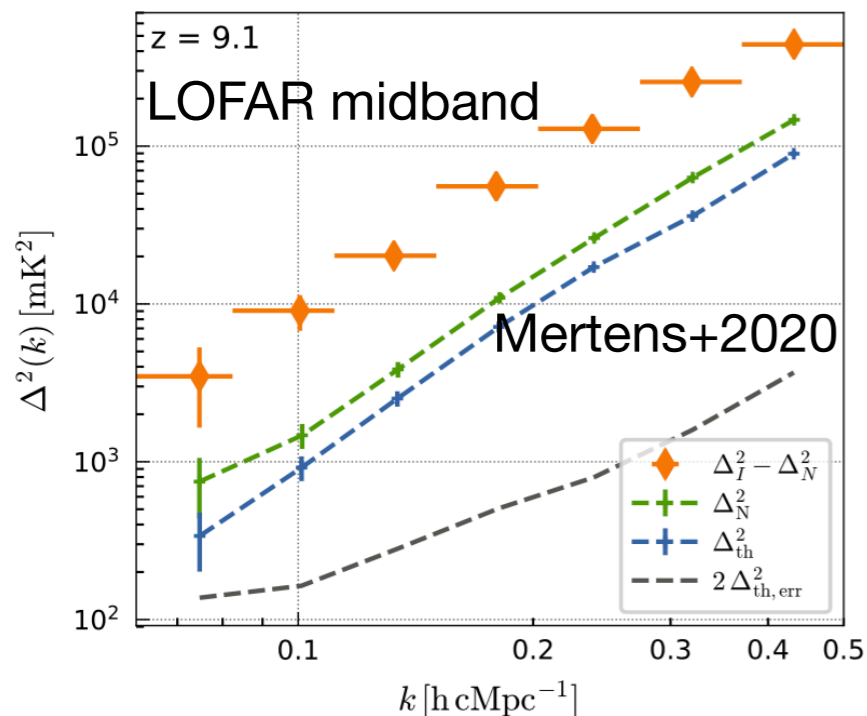
Promising technique for foreground subtraction



Gaussian process regression for FG subtraction (GPR-FS)

Promising technique for foreground subtraction

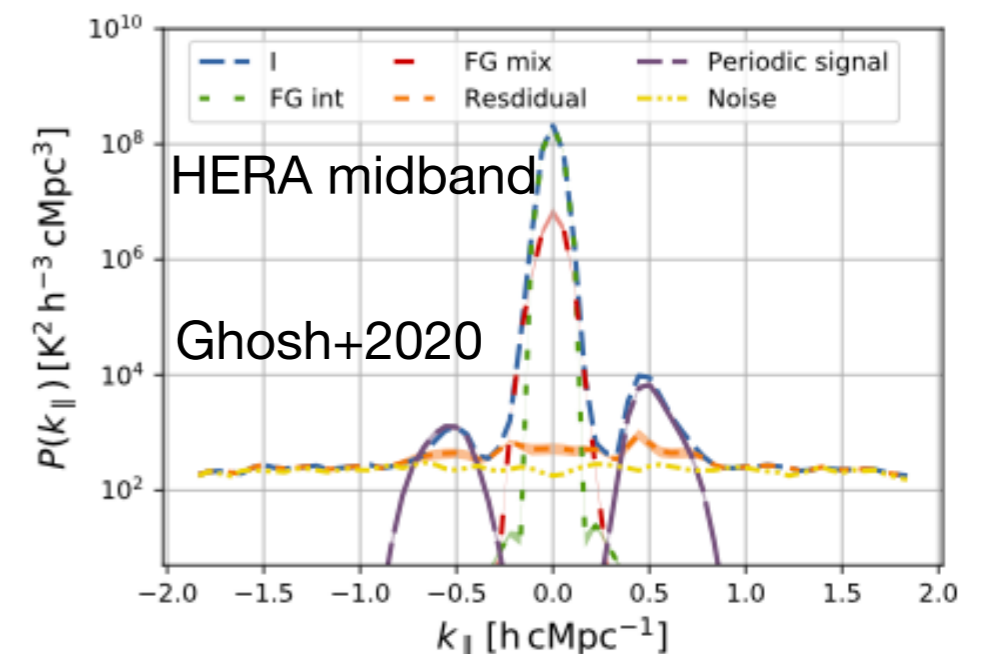
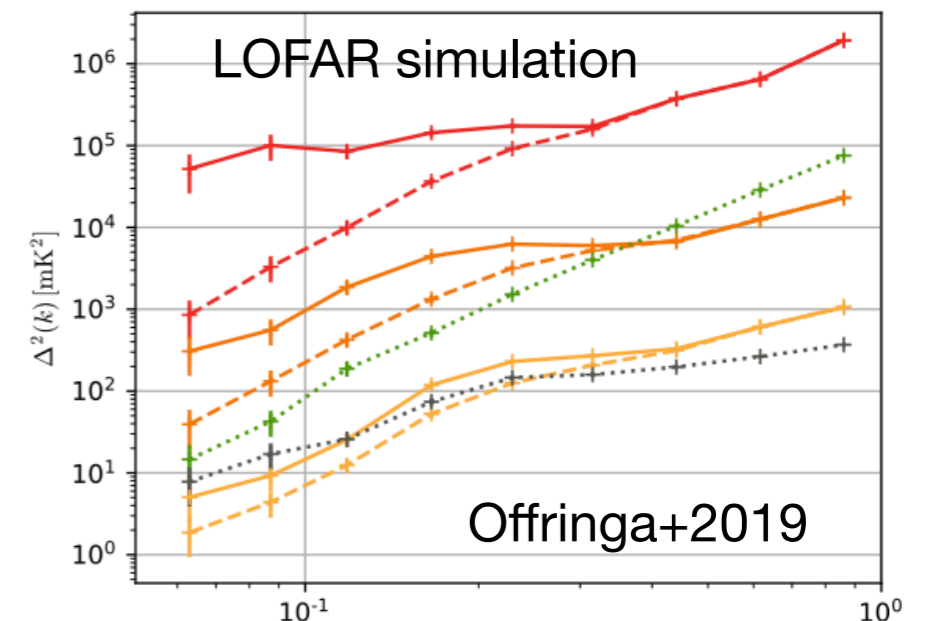
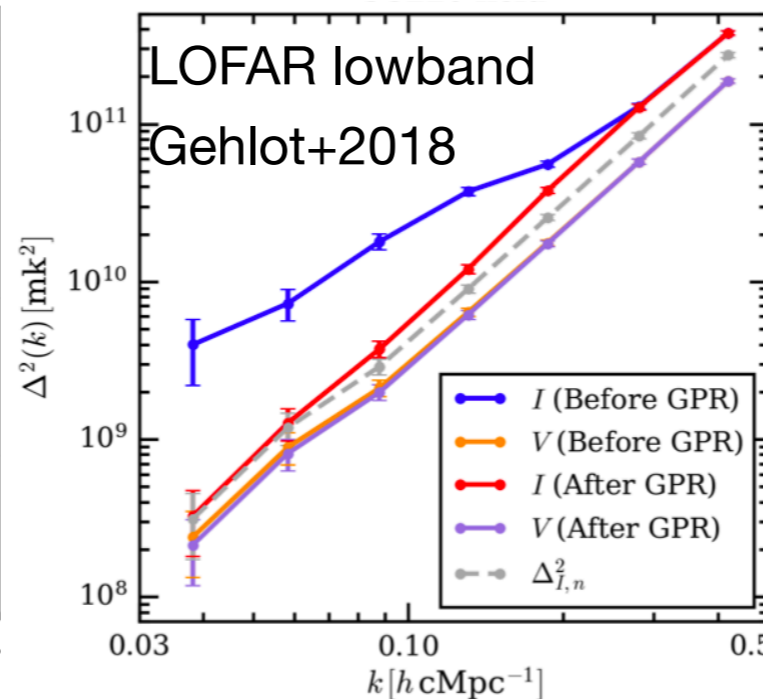
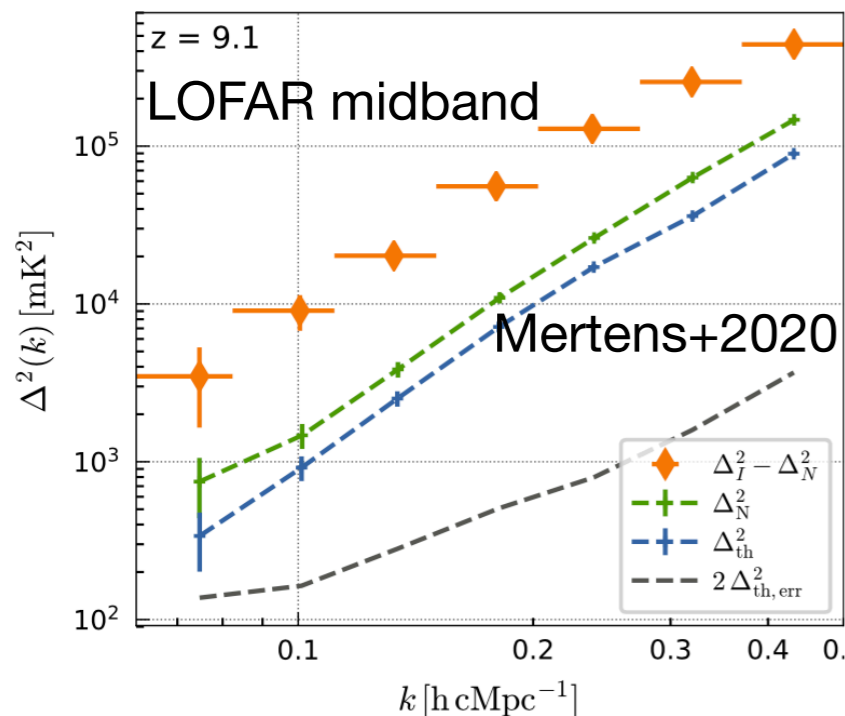
Becoming more widely
used in 21 cm analyses



Gaussian process regression for FG subtraction (GPR-FS)

Promising technique for foreground subtraction

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Gaussian process regression for FG subtraction (GPR-FS)

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Why is it so effective?

How does it compare to existing covariance-
based techniques?

GPR-FS: How it works

Condition the joint density on the data

$$f|d \sim \mathcal{N}(\mathbf{E}[f], \text{Cov}[f])$$

$$\mathbf{E}[f] = C_{fg} C^{-1} d$$

GPR-FS: How it works

Condition the joint density on the data

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Form the residual! $r = d - \mathbf{E}[f]$

GPR-FS: How it works

Condition the joint density on the data

$$f|d \sim \mathcal{N}(\mathbf{E}[f], \text{Cov}[f])$$

$$\mathbf{E}[f] = C_{\text{fg}} C^{-1} d$$

Form the residual!

$$r = d - \mathbf{E}[f]$$

$$r = (I - C_{\text{fg}} C^{-1}) d$$

$$r = R_{\text{GPR-FS}} d$$

Inverse covariance weighting and the OQE

A general quadratic estimator (QE) of the power spectrum

Inverse covariance weighting and the OQE

A general quadratic estimator (QE) of the power spectrum

$$d = \begin{bmatrix} d_{\nu_1} \\ d_{\nu_2} \\ \vdots \end{bmatrix} \quad \hat{q}_\alpha = d^T R^T C_{,\alpha} R d$$

Fourier transform operator

weighting matrix

Inverse covariance weighting and the OQE

A general quadratic estimator (QE) of the power spectrum

$$d = \begin{bmatrix} d_{\nu_1} \\ d_{\nu_2} \\ \vdots \end{bmatrix} \quad \hat{q}_\alpha = d^T R^T C_{,\alpha} R d$$

Fourier transform operator

weighting matrix

The optimal quadratic estimator (OQE)

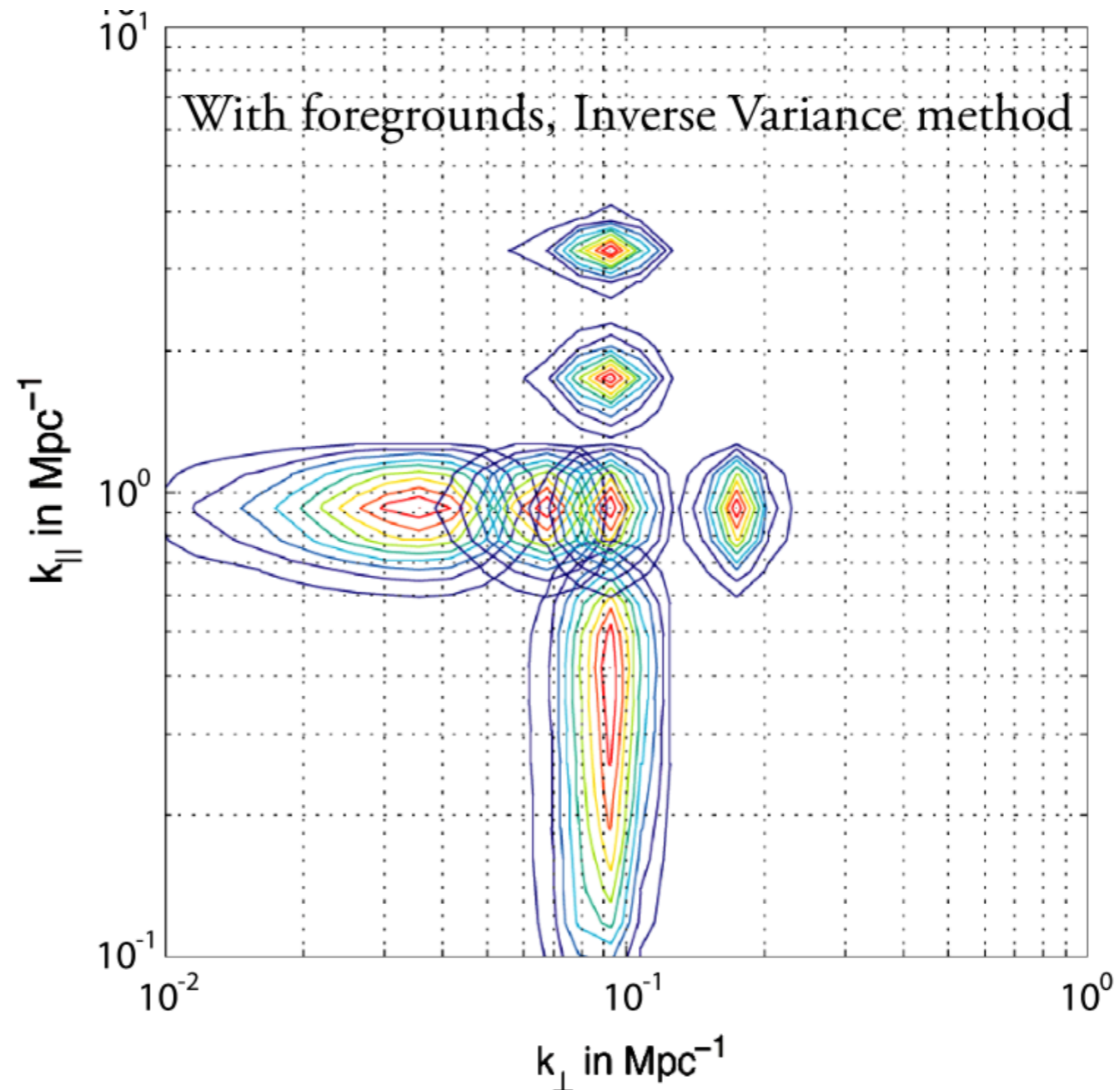
$$R = C^{-1}$$

$$C = C_{\text{fg}} + C_{\text{s}} + C_{\text{n}}$$

The OQE for 21 cm: caveats

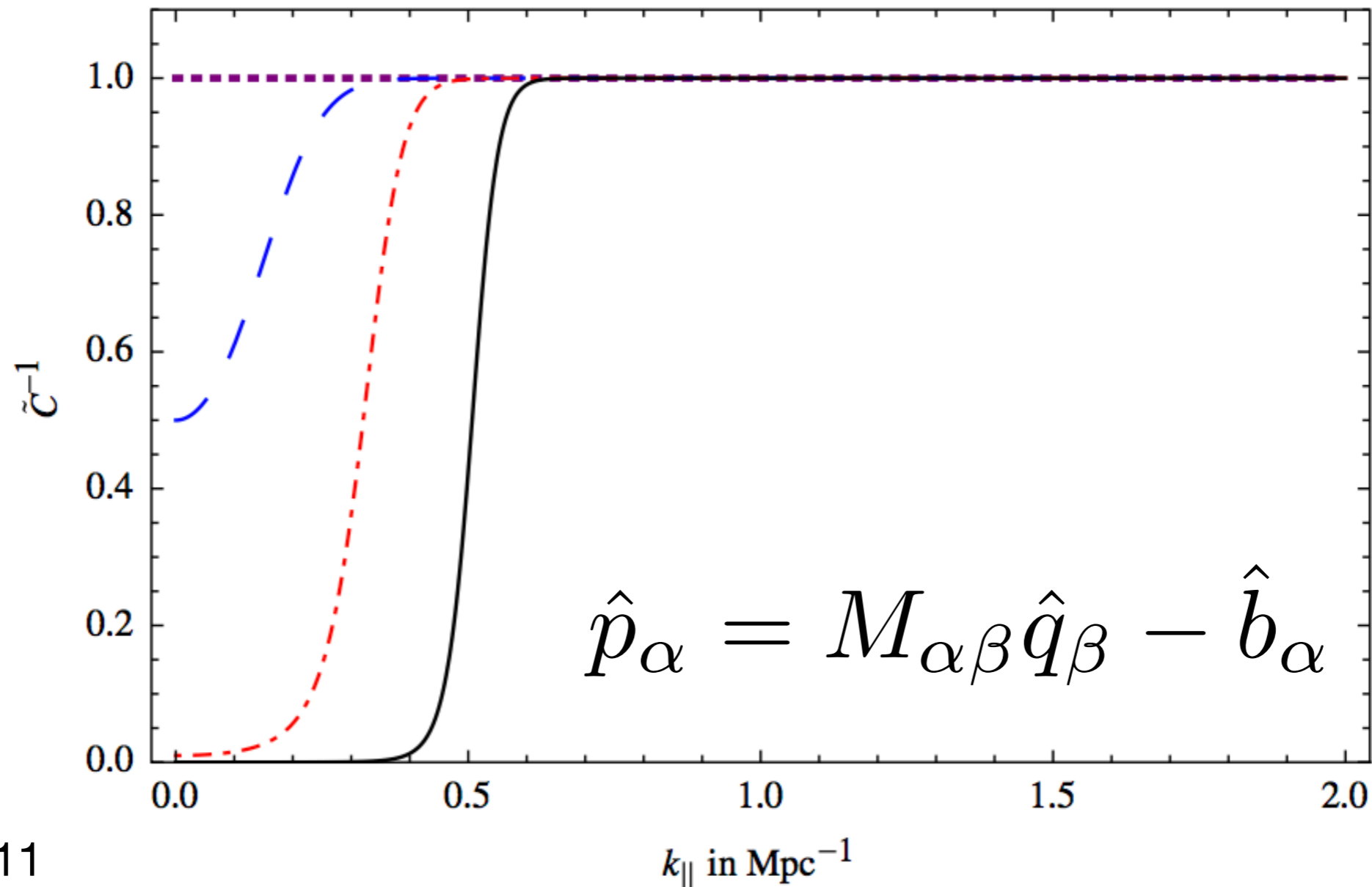
1. Window functions can be non-trivial!

$$\langle \hat{p} \rangle = W p_{\text{true}}$$



The OQE for 21 cm: caveats

2. Inverse covariance is a “high-pass filter”, requires normalization



The OQE for 21 cm: caveats

3. Residual bias subtraction is tricky

$$\hat{p}_\alpha = M_{\alpha\beta} \hat{q}_\beta - \hat{b}_\alpha$$

$$\hat{b}_\alpha = \text{tr}[(N + C_{\text{fg}}) M_{\alpha\beta} C_{,\beta}]$$

need to know this very accurately



GPR-FS and the OQE

How does GPR-FS relate to the OQE?

GPR-FS and the OQE

How does GPR-FS relate to the OQE?

$$C^{-1} = [C_n + C_{HI}]^{-1} (I - C_{fg} C^{-1})$$

GPR-FS and the OQE

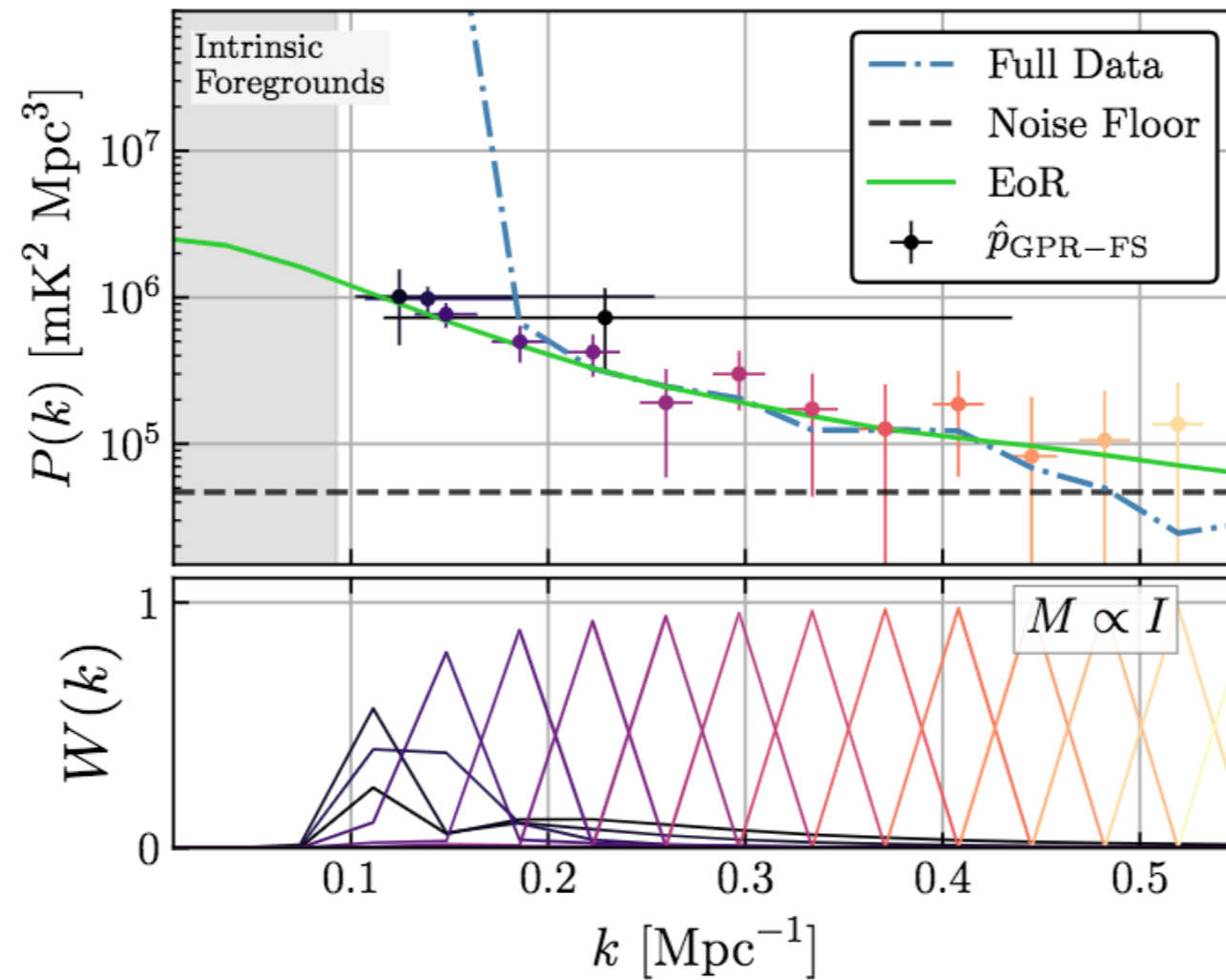
How does GPR-FS relate to the OQE?

$$C^{-1} = [C_n + C_{\text{HI}}]^{-1} (I - C_{\text{fg}} C^{-1})$$

$$R_{\text{OQE}} = [C_n + C_{\text{HI}}]^{-1} R_{\text{GPR-FS}}$$

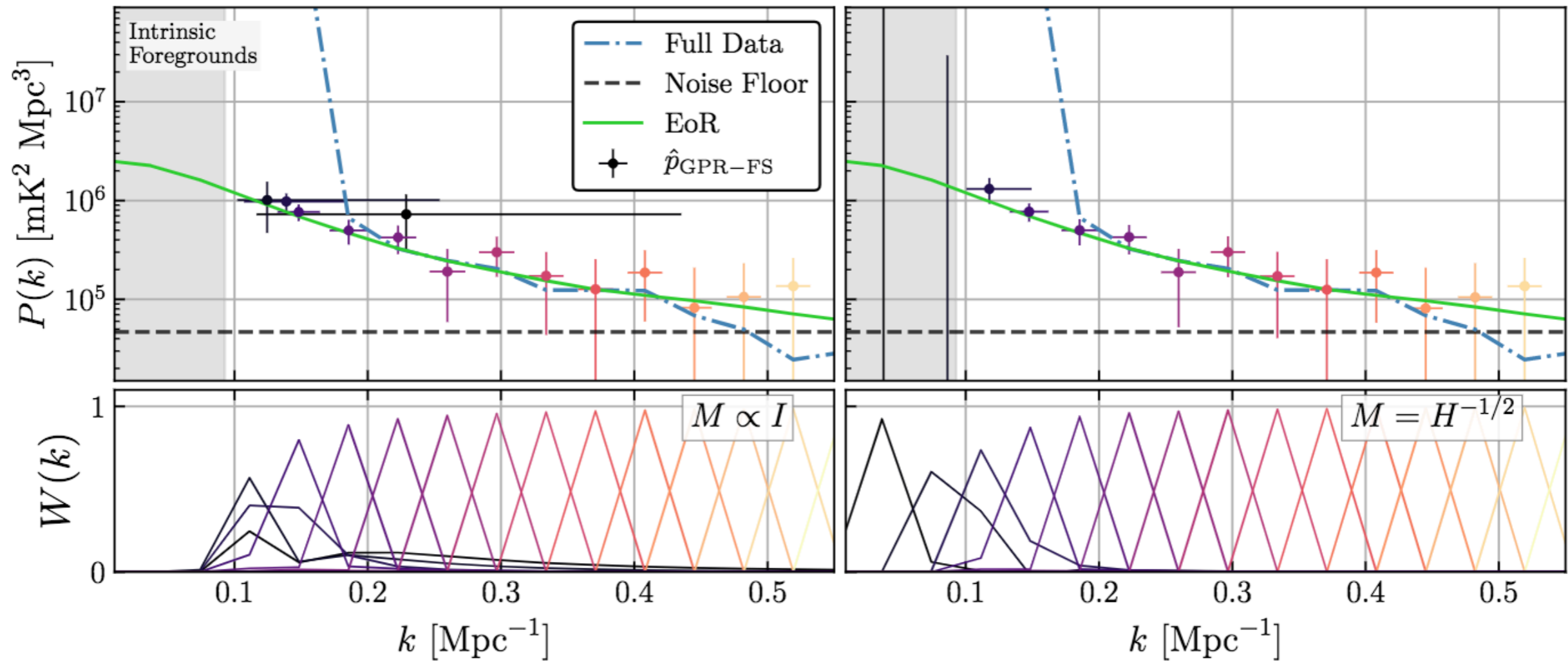
GPR-FS on mock data

Mapping the window functions

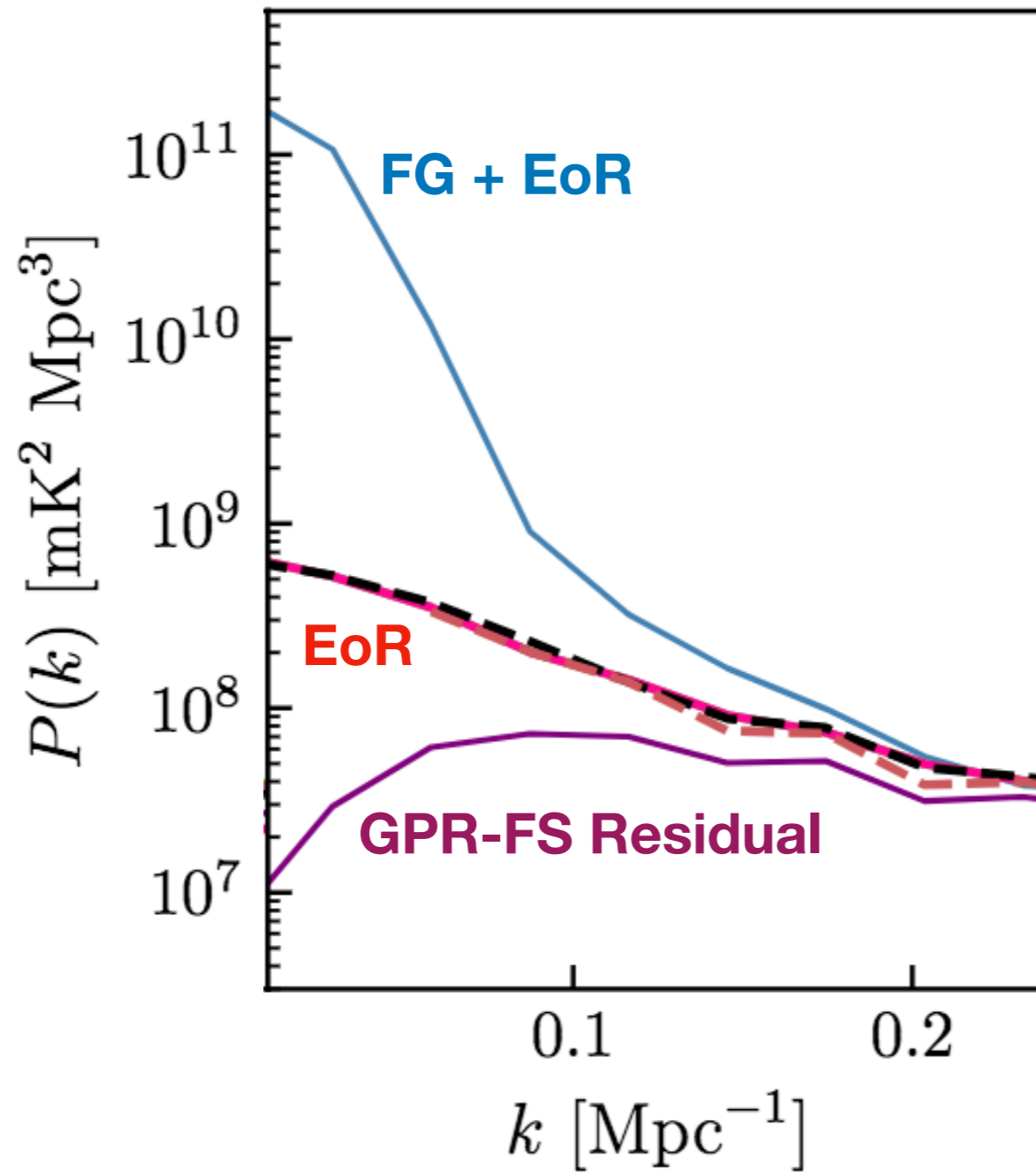


GPR-FS on mock data

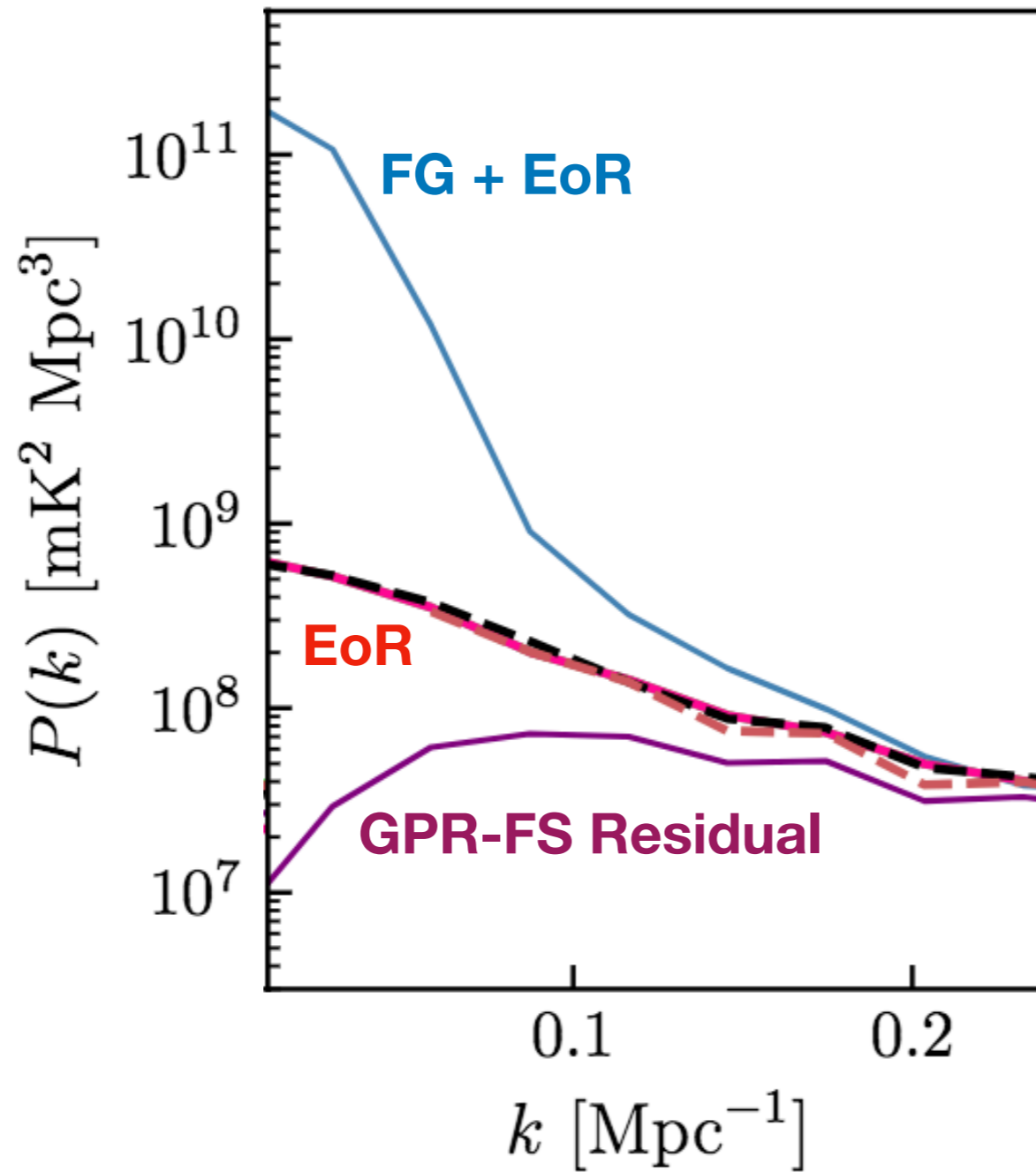
Mapping the window functions



GPR-FS *requires* normalization



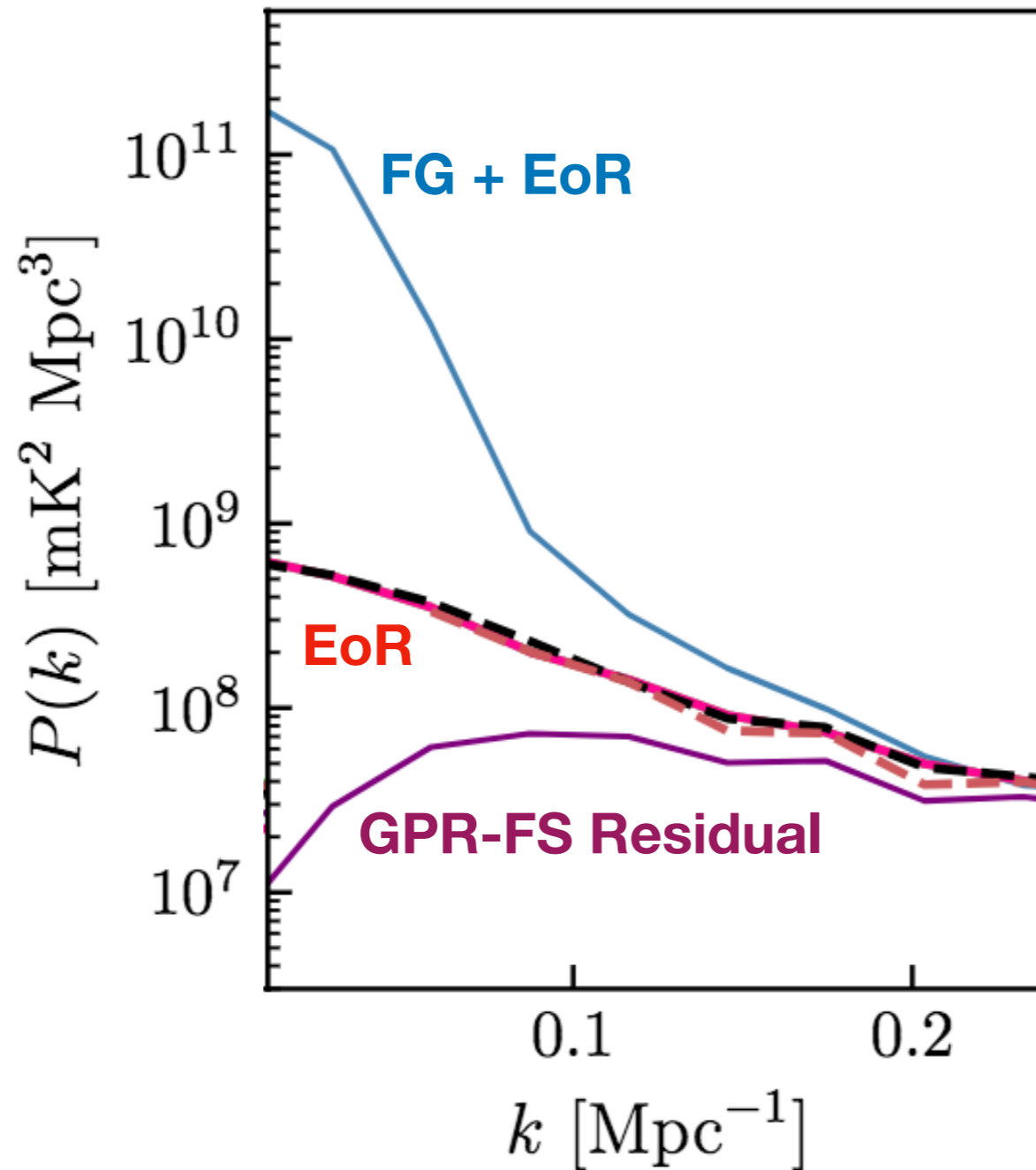
GPR-FS *requires* normalization



$$\hat{p}_\alpha = M_{\alpha\beta} \hat{q}_\beta - \hat{b}_\alpha$$

■ ■

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$$\hat{p}_\alpha = M_{\alpha\beta} \hat{q}_\beta - \hat{b}_\alpha$$

■ ■

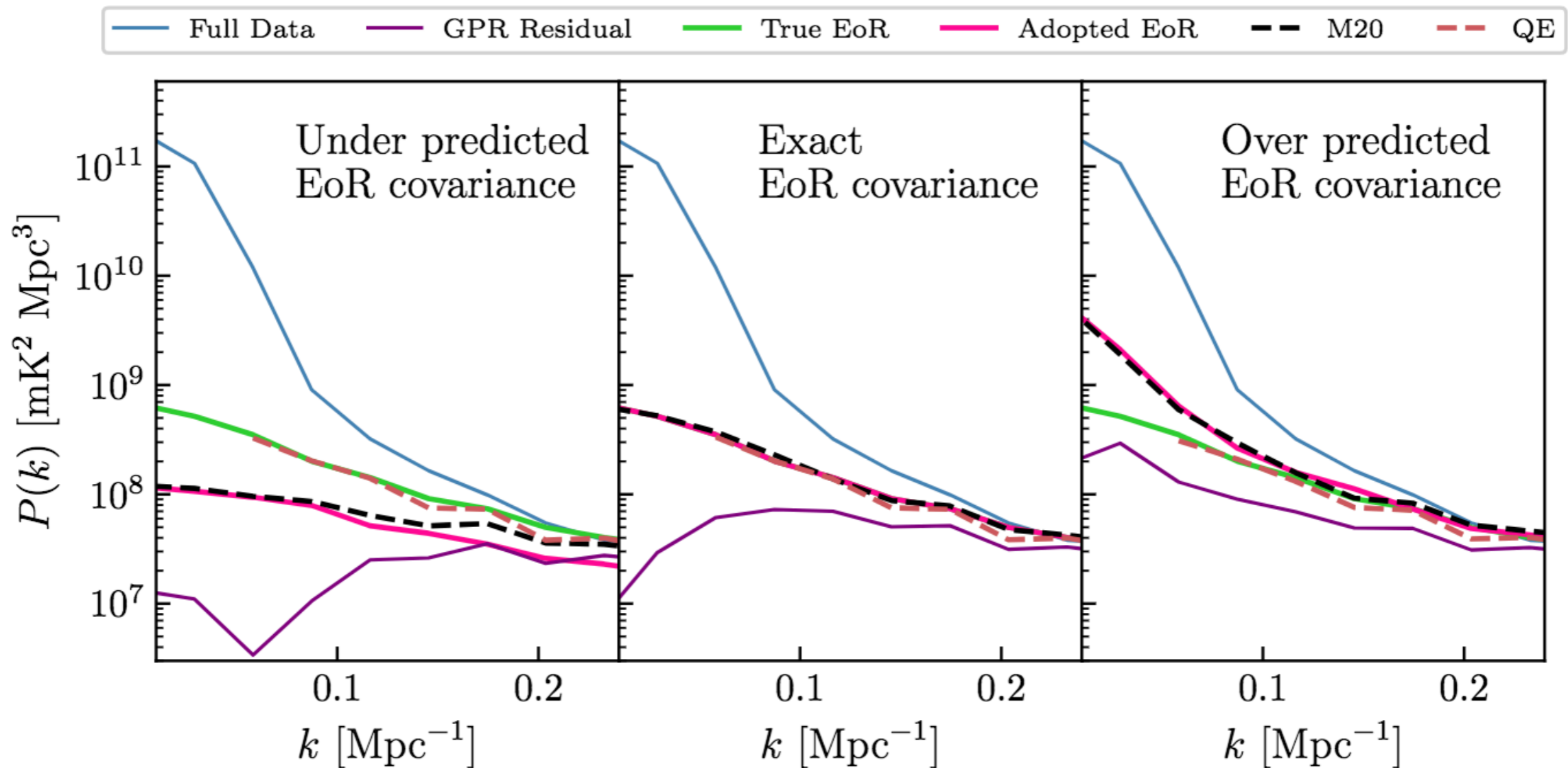
Recognized by
Mertens+2020, but
treated differently

The LOFAR GPR-FS pipeline

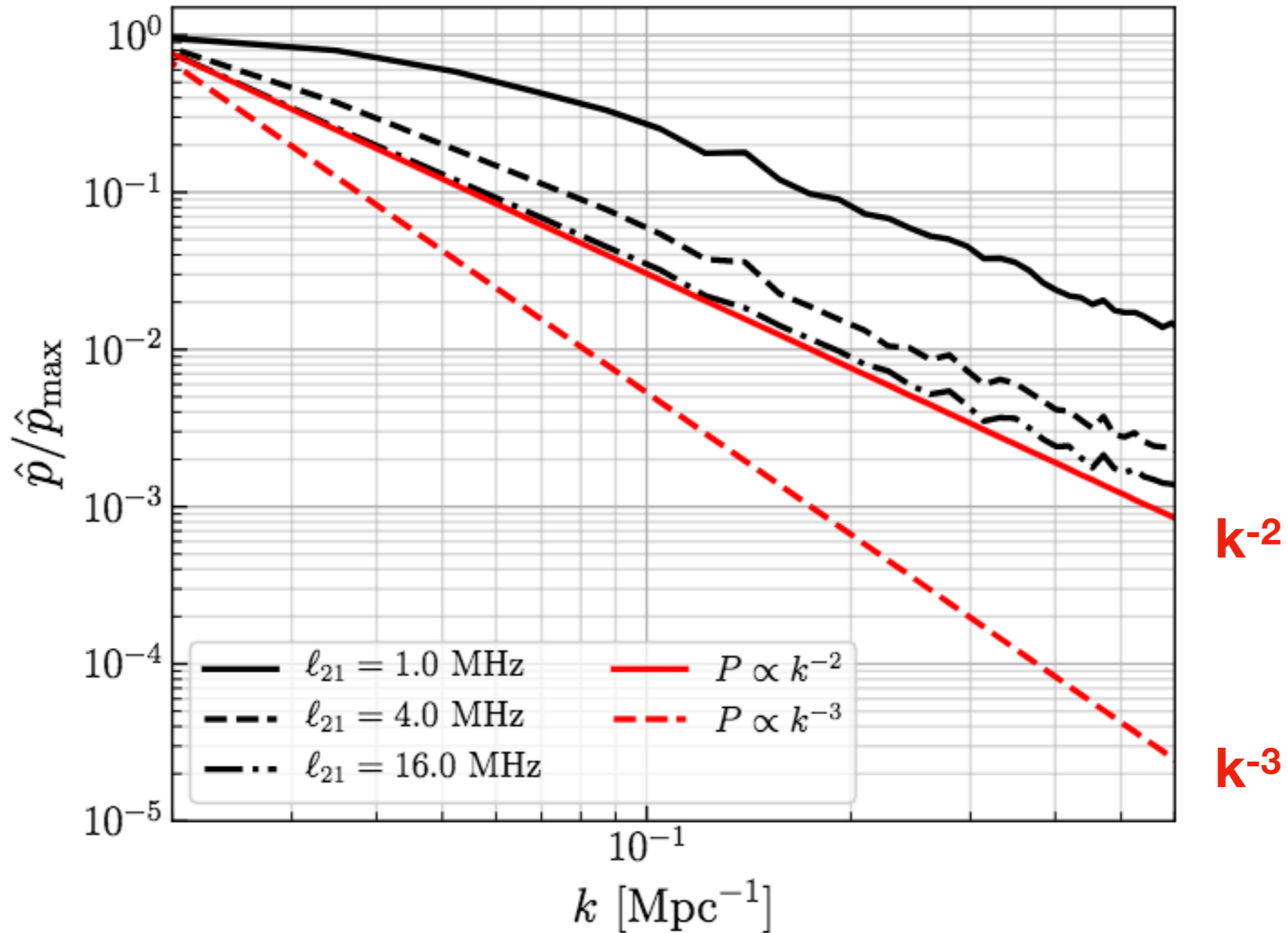
LOFAR normalizes their post GPR-FS data with a **bias correction**

The LOFAR GPR-FS pipeline

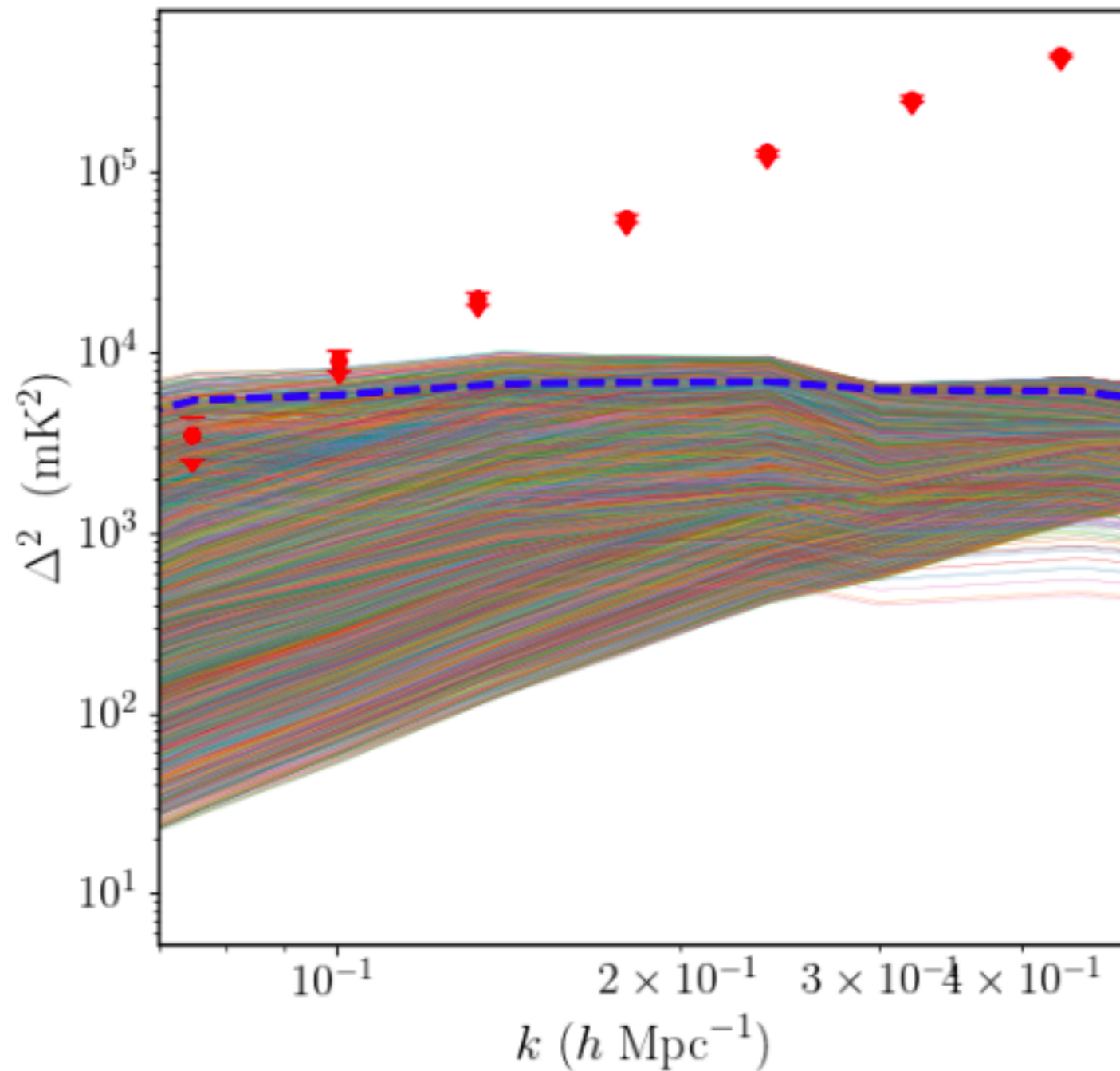
LOFAR normalizes their post GPR-FS data with a **bias correction**



Results only as good as the EoR model



May be an oversight for ruling out certain models



Ruling out these kinds of models may be problematic

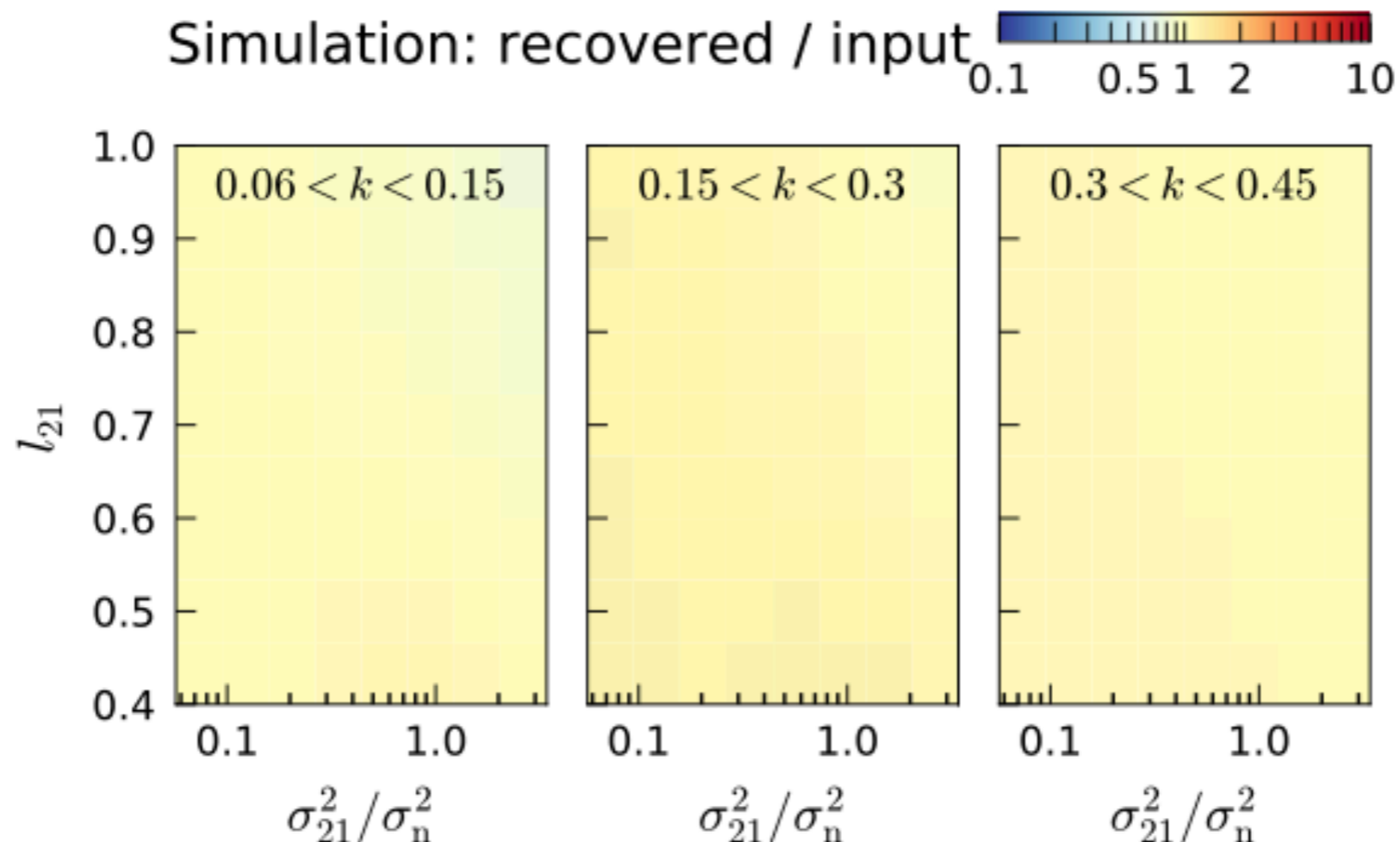
$$P(k) \propto k^{-3} \Delta^2$$

LOFAR results do acknowledge this

- Test a few different covariance models before settling on an exponential EoR covariance

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- Test a few different covariance models before settling on an exponential EoR covariance
- Simulated power spectrum recovery tests

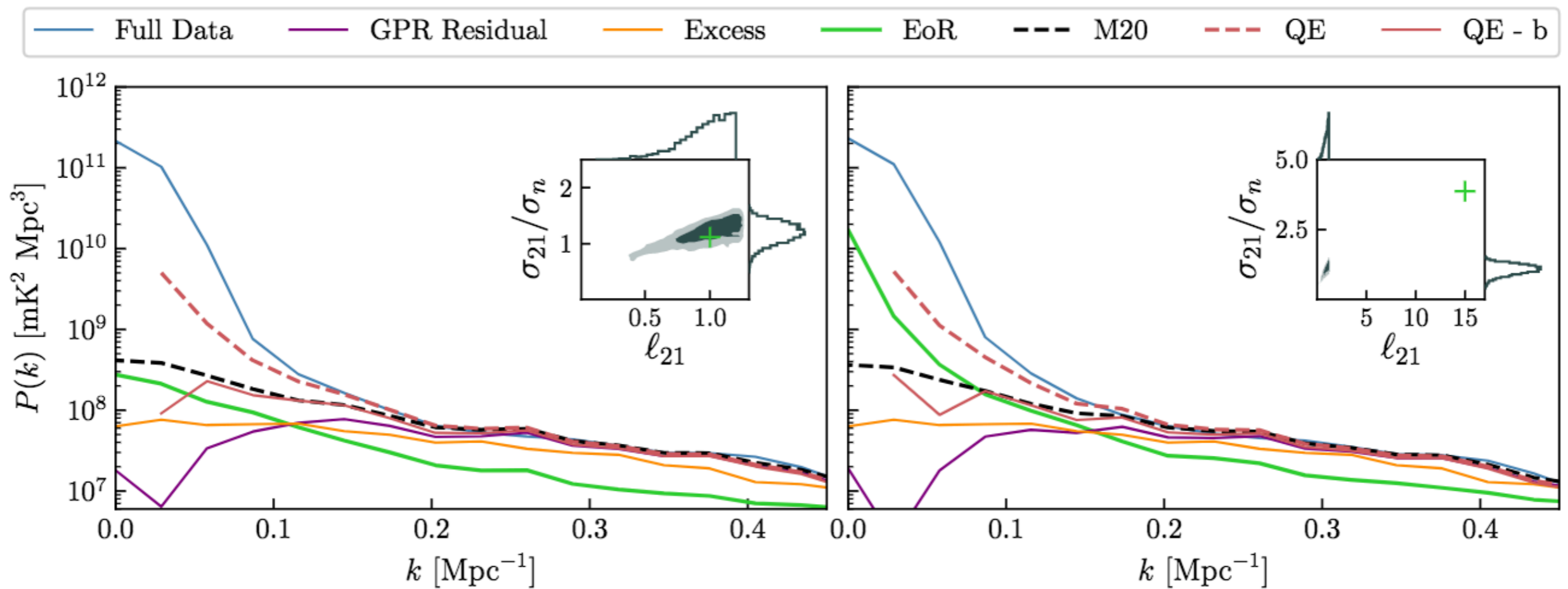


Summary

1. Gaussian process foreground subtraction is closely related to inverse covariance weighting
2. Window functions important for low k recovery
3. Current LOFAR estimator is particularly sensitive to a mismatched EoR covariance

Questions? Let's chat on the SALF slack! (@nkern)

Signal Recovery Test I: Impact of prior



Signal Recovery Test II: Tone injection

